ＴＥＸＴ　ＢＯＯＫ１

ARAB MATHEMATICS:ＯＭＡＲ AL-ＫＨＡYYAM

－Ａ　ＣＵＢＥ　ＡＮＤ　ＳＩＤＥＳ　ＡＲＥ　ＥＱＵＡＬ　ＴＯ ＮＵＭＢＥＲ－

授業者　ＭＳＴ．ＺＩＡＵＮ　ＮＡＨＡＲ
（筑波大学大学院教育研究科教育研究科教科教育専攻数学教育コース）

① WHO IS OMAR

NAME:-

Omar Ibn Ibrahim al Khayyam, Giyat-ed-din Abul Fath.

He was born 1044 A.D. at NISHABUR in IRAN.

He died 1123 A.D.

The name al-Khayyam in Arabic means the tentmaker.

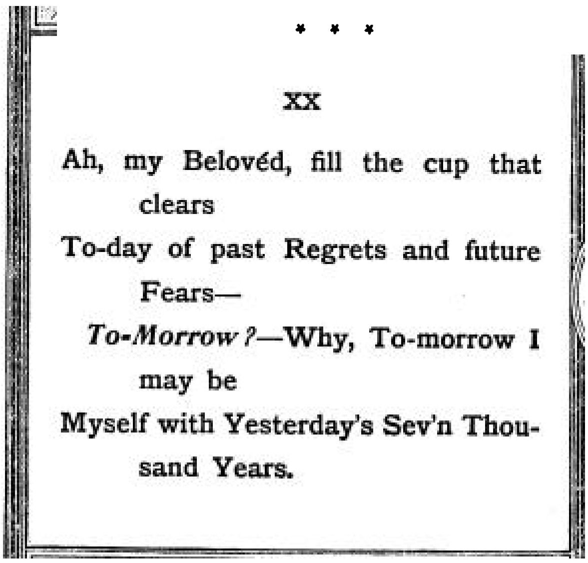
Omar himself never practiced this profession but it was the occupation of his father or one of his ancestors; hence the family name.

2 RUBAIYAT OF OMAR KHAYYAM

OMAR AS A BELOVED POET :-

He wrote “ RUBAIAT OF OMAR KHAYYAM “, it is literary of poem.
It should be appreciated that it is practically impossible to exactly translate any literary work into another language, what to talk of poetry, especially when it involves mystical messages of deep complexity. Despite this, the popularity of the translation of RUBAIYAT would indicate the wealth of his rich thought.

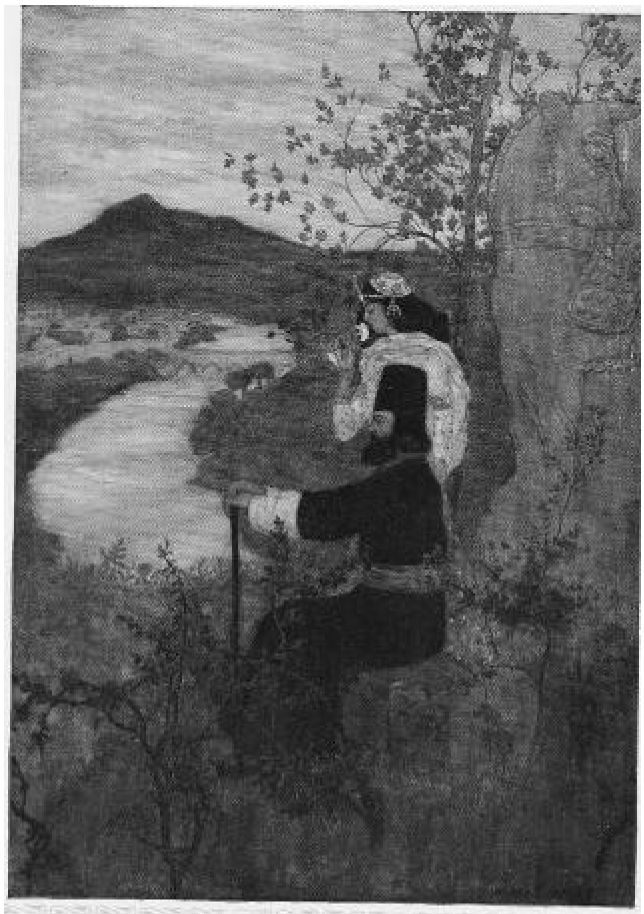
برچهره گل شبنم نوروز خوش است   در طرف چمن روی دل افروز خوش است
از دی که گذشت هرچه گویی خوش نیست   خوش باش وز دی مگو که امروز خوش است



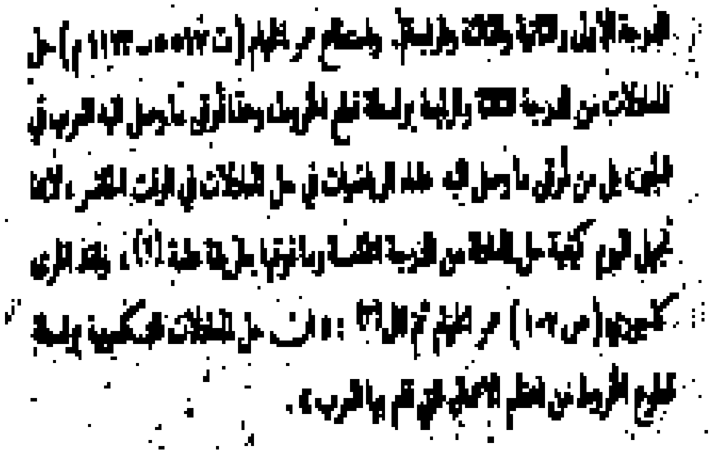
* * *

XX

Ah, my Belovéd, fill the cup that clears
To-day of past Regrets and future Fears—
To-Morrow ?—Why, To-morrow I may be
Myself with Yesterday's Sev'n Thousand Years.



Original Arabic and translation



OMAR-AL-KHAYYAM could solve three degree equation from third and fourth term by using geometrical shape, and this is the highest peak (top) reached by ARAB's in algebra and it is higher than what some mathematician have reached now-a-days, because still now we can not solve equations from fifth degree or higher by a common way. Gregory had praised Omar –al-Khayyam, and said, "solving cubic equations by using geometrical representation, was one of the greatest achievements of ARAB scientists".

A CUBE AND SIDES ARE EQUAL TO A NUMBER
3—1 PROPOSITION 14 (Book 2)

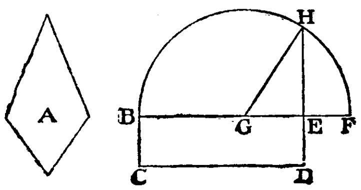
PROPOSITION 14. *PROBLEM*

To describe a square that shall be equal to a given rectilineal figure.

Let *A* be the given rectilineal figure: it is required to describe a square that shall be equal to *A*.

Describe the rectangular parallelogram *BCDE* equal to the rectilineal figure *A*. [I. 45.

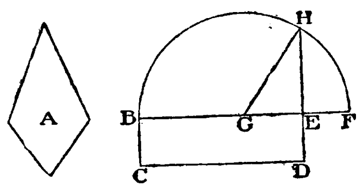
Then if the sides of it, *BE*, *ED*, are equal to one another, it is a square, and what was required is now done.



But if they are not equal, produce one of them *BE* to *F*, make *EF* equal to *ED*, [I. 3.

and bisect *BF* at *G*; [I. 10.

from the centre *G*, at the distance *GB*, or *GF*, describe the semicircle *BHF*, and produce *DE* to *H*.



The square described on *EH* shall be equal to the given rectilineal figure *A*.

Join *GH*. Then, because the straight line *BF* is divided into two equal parts at the point *G*, and into two unequal parts at the point *E*, the rectangle *BE*, *EF*, together with the square on *GE*, is equal to the square on *GF*. [II. 5.

But *GF* is equal to *GH*.

Therefore the rectangle *BE*, *EF*, together with the square on *GE*, is equal to the square on *GH*.

But the square on *GH* is equal to the squares on *GE*, *EH*; [I. 47.

therefore the rectangle *BE*, *EF*, together with the square on *GE*, is equal to the squares on *GE*, *EH*.

Take away the square on *GE*, which is common to both;

therefore the rectangle *BE*, *EF* is equal to the square on *EH*. [*Axiom* 3.

But the rectangle contained by *BE*, *EF* is the parallelogram *BD*.

because *EF* is equal to *ED*. [*Construction*.

Therefore *BD* is equal to the square on *EH*.

But *BD* is equal to the rectilineal figure *A*. [*Construction*.

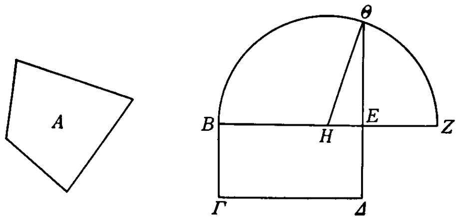
Therefore the square on *EH* is equal to the rectilineal figure *A*.

Wherefore *a square has been made equal to the given rectilineal figure A, namely, the square described on EH.* Q.E.F.

14

与えられた直線図形に等しい正方形をつくること。

与えられた直線図形を A とせよ。このとき直線図形 A に等しい正方形をつくらねばならぬ。



直線図形 A に等しい直角平行四辺形 $B\Delta$ がつくられたとせよ。そうすればもし BE が $E\Delta$ に等しければ，命じられたことはなされたことになるであろう。なぜなら正方形 $B\Delta$ が直線図形 A に等しくつくられたから。もし等しくなければ，BE, $E\Delta$ の一方が大きい。BE が大きいとし，BE が Z まで延長され，EZ が $E\Delta$ に等しくされ，BZ が H で 2 等分され，H を中心とし，HB, HZ の一を半径として半円 $B\Theta Z$ が描かれ，ΔE が Θ まで延長され，$H\Theta$ が結ばれたとせよ。

そうすれば線分 BZ は H において等しい部分に，E において不等な部分に分けられたから，BE, EZ にかこまれた矩形と EH 上の正方形との和は HZ 上の正方形に等しい。そして HZ は $H\Theta$ に等しい。それゆえ矩形 BE, EZ と HE 上の正方形との和は $H\Theta$ 上の正方形に等しい。ところが ΘE, EH 上の正方形の和は $H\Theta$ 上の正方形に等しい。ゆえに矩形 BE, EZ と HE 上の正方形との和は ΘE, EH 上の正方形の和に等しい。双方から HE 上の正方形がひかれたとせよ。そうすれば残りの BE, EZ にかこまれた矩形は $E\Theta$ 上の正方形に等しい。ところが EZ は $E\Delta$ に等しいから，矩形 BE, EZ は $B\Delta$ である。それゆえ平行四辺形 $B\Delta$ は ΘE 上の正方形に等しい。そして $B\Delta$ は直線図形 A に等しい。ゆえに直線図形 A も $E\Theta$ 上に描かれた正方形に等しい。

よって与えられた直線図形 A に等しい正方形，すなわち $E\Theta$ 上に描かれうる正方形がつくられた。これが作図すべきものであった*)。

3−3 A CUBE AND SIDES ARE EQUAL TO NUMBER

How To Solve The Problem Of OmarKhayyaam

I. The first species. *A cube and sides are equal to a number.*[1]

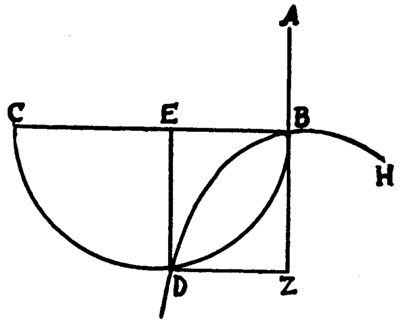


FIG. 17

Let the line AB (Fig. 17) be the side of a square equal to the given number of roots.[2] Construct a solid whose base is equal to the square on AB, equal in volume to the given number. The construction has been shown previously.[3] Let BC be the height of the solid. Let BC be perpendicular to AB. You know already what meaning is applied in this discussion to the phrase *solid number*. It is a solid whose base is the square of unity and whose height is equal to the given number; that is, the height is a line whose ratio to the side of the base of the solid is as the ratio of the given number to one. Produce AB to Z and construct a parabola whose vertex is the point B, axis BZ, and parameter AB. Then the position of the conic HBD will be known, as has been shown previously and it will be tangent to BC. Describe on BC a semicircle. It necessarily intersects the conic. Let the point of intersection be D; drop from D, whose position is known, two perpendiculars DZ and DE on BZ and BC. Both the position and the magnitude of these lines are known. The line DZ is an ordinate of the conic. Its square is then equal to the product of BZ and AB.[5] Consequently, AB to DZ, which is equal to BE, is as BE to ED, which is equal to ZB.[6] But BE to ED is as ED to EC.[7] The four lines then are in continuous proportion, AB, BE, ED, EC,[8] and consequently the square of the parameter AB, the first, is to the square of BE, the second, as BE, the second, is to EC, the fourth.[9] The solid whose base is the square AB and whose height is EC is equal to the cube BE, because the heights of these figures are reciprocally equal to their bases.[10] Let the solid whose base is the square of AB and height is EB be added to both.[11] The cube BE plus the solid then is equal to the solid whose base is the square AB and whose height is BC, which solid we have assumed to be equal to the given number. But the solid whose base is the square of AB, which is equal to the number of roots, and whose height is EB, which is the side of the cube, is equal to the number of the given sides of the cube EB. The cube EB, then, plus the number of its given sides is equal to the given number, which was required.

This species does not present varieties of cases or impossible problems. It has been solved by means of the properties of the circle combined with those of the parabola.

II. The second species.