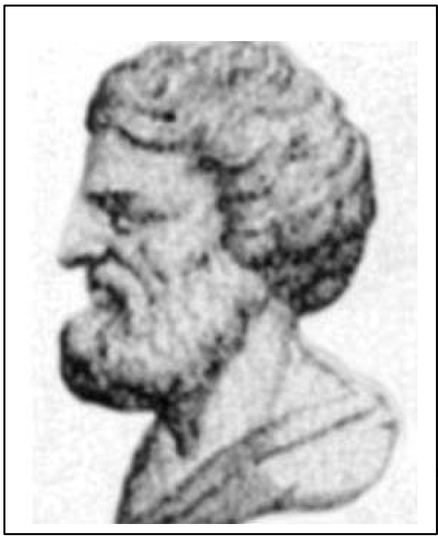
2002年10月30日（水）

授業資料

日時計と円錐曲線について

研究授業３日目



Apollonius

２年　　組　　番

名前＿＿＿＿＿＿＿＿＿＿＿＿＿＿

授業者：筑波大学大学院教育研究科　高見香織

復習

比例中項の関係

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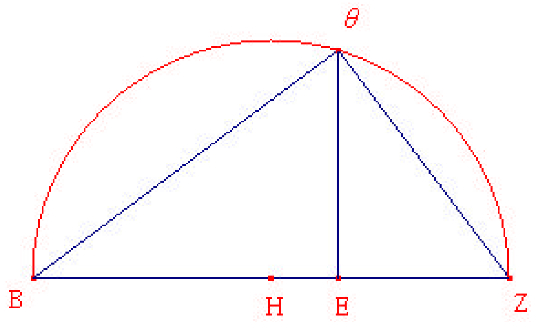


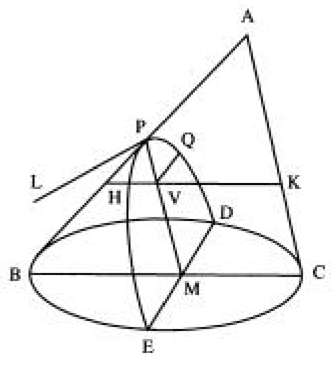
命題 11

頂点が点 A で、底面が BC を直径とする円である円錐について考える。軸を通る面で切断し、できた切断面を三角形 ABC とする。その三角形の底辺 BC に垂直かつ一方の母線 AC に平行となる面で切断し、その切り口を EPD とする。点 P から切り口（PM）に垂直になるように PL を描き、その長さを

PL：PA＝BC^2：BA・AC

となるようにする。点 Q をその切り口上に任意に取り、点 Q を通り、DE に平行に QV をひく。

このとき、QV^2＝PL・PV　であるといえる。



１．アポロニウスが考えた円錐曲線の性質について

基本的な性質について　【引用文献：Greek Mathematics Work Ⅰ　IVOR THOMAS 著】

ギリシア語

ιβ′

'Εὰν κῶνος ἐπιπέδῳ τμηθῇ διὰ τοῦ ἄξονος, τμηθῇ δὲ καὶ ἑτέρῳ ἐπιπέδῳ τέμνοντι τὴν βάσιν

τοῦ κώνου κατ' εὐθεῖαν πρὸς ὀρθὰς οὖσαν τῇ βάσει τοῦ διὰ τοῦ ἄξονος τριγώνου, καὶ ἡ διάμετρος τῆς τομῆς ἐκβαλλομένη συμπίπτῃ μιᾷ πλευρᾷ τοῦ διὰ τοῦ ἄξονος τριγώνου ἐκτὸς τῆς τοῦ κώνου κορυφῆς, ἥτις ἂν ἀπὸ τῆς τομῆς ἀχθῇ παράλληλος τῇ κοινῇ τομῇ τοῦ τέμνοντος ἐπιπέδου καὶ τῆς βάσεως τοῦ κώνου, ἕως τῆς διαμέτρου τῆς τομῆς δυνήσεταί τι χωρίον παρακείμενον παρά τινα εὐθεῖαν, πρὸς ἣν λόγον ἔχει ἡ ἐπ' εὐθείας μὲν οὖσα τῇ διαμέτρῳ τῆς τομῆς, ὑποτείνουσα δὲ τὴν ἐκτὸς τοῦ τριγώνου γωνίαν, ὃν τὸ τετράγωνον τὸ ἀπὸ τῆς ἠγμένης ἀπὸ τῆς κορυφῆς τοῦ κώνου παρὰ τὴν διάμετρον τῆς τομῆς ἕως τῆς βάσεως τοῦ τριγώνου πρὸς τὸ περιεχόμενον ὑπὸ τῶν τῆς βάσεως τμημάτων, ὧν ποιεῖ ἡ ἀχθεῖσα, πλάτος ἔχον τὴν ἀπολαμβανομένην ὑπ' αὐτῆς ἀπὸ τῆς διαμέτρου πρὸς τῇ κορυφῇ τῆς τομῆς, ὑπερβάλλον εἴδει ὁμοίῳ τε καὶ ὁμοίως κειμένῳ τῷ περιεχομένῳ ὑπό τε τῆς ὑποτεινούσης τὴν ἐκτὸς γωνίαν τοῦ τριγώνου καὶ τῆς παρ' ἣν δύνανται αἱ καταγόμεναι· καλείσθω δὲ ἡ τοιαύτη τομὴ ὑπερβολή.

Ἔστω κῶνος, οὗ κορυφὴ μὲν τὸ Α *σημεῖον, βάσις δὲ ὁ* ΒΓ *κύκλος, καὶ τετμήσθω ἐπιπέδῳ διὰ τοῦ ἄξονος, καὶ ποιείτω τομὴν τὸ* ΑΒΓ *τρίγωνον, τετμήσθω δὲ καὶ ἑτέρῳ ἐπιπέδῳ τέμνοντι τὴν βάσιν τοῦ κώνου κατ' εὐθεῖαν τὴν* ΔΕ *πρὸς ὀρθὰς οὖσαν τῇ* ΒΓ *βάσει τοῦ* ΑΒΓ *τριγώνου, καὶ ποιείτω τομὴν ἐν τῇ ἐπιφανείᾳ τοῦ κώνου τὴν* ΔΖΕ *γραμμήν, ἡ δὲ διάμετρος τῆς τομῆς ἡ* ΖΗ *ἐκβαλλομένη συμπιπτέτω μιᾷ πλευρᾷ τοῦ* ΑΒΓ *τριγώνου τῇ* ΑΓ *ἐκτὸς τῆς τοῦ κώνου κορυφῆς κατὰ τὸ* Θ, *καὶ διὰ τοῦ* Α *τῇ διαμέτρῳ τῆς τομῆς*

τῇ ΖΗ *παράλληλος ἤχθω ἡ* ΑΚ, *καὶ τεμνέτω τὴν* ΒΓ, *καὶ ἀπὸ τοῦ* Ζ *τῇ* ΖΗ *πρὸς ὀρθὰς ἤχθω ἡ*

ΖΛ, *καὶ πεποιήσθω, ὡς τὸ ἀπὸ* ΚΑ *πρὸς τὸ ὑπὸ* ΒΚΓ, *οὕτως ἡ* ΖΘ *πρὸς* ΖΛ, *καὶ εἰλήφθω τι σημεῖον ἐπὶ τῆς τομῆς τυχὸν τὸ* Μ, *καὶ διὰ τοῦ* Μ *τῇ* ΔΕ *παράλληλος ἤχθω ἡ* ΜΝ, *διὰ δὲ τοῦ* Ν *τῇ* ΖΛ *παράλληλος ἡ* ΝΟΞ, *καὶ ἐπιζευχθεῖσα ἡ* ΘΛ *ἐκβεβλήσθω ἐπὶ τὸ* Ξ, *καὶ διὰ τῶν* Λ, Ξ *τῇ* ΖΝ *παράλληλοι ἤχθωσαν αἱ* ΛΟ, ΞΠ. *λέγω, ὅτι ἡ* ΜΝ *δύναται τὸ* ΖΞ, *ὃ παράκειται παρὰ τὴν* ΖΛ, *πλάτος ἔχον τὴν* ΖΝ, *ὑπερβάλλον εἴδει τῷ* ΛΞ *ὁμοίῳ ὄντι τῷ ὑπὸ τῶν* ΘΖΛ.

Ἤχθω γὰρ διὰ τοῦ Ν *τῇ* ΒΓ *παράλληλος ἡ* ΡΝΣ· *ἔστι δὲ καὶ ἡ* ΝΜ *τῇ* ΔΕ *παράλληλος· τὸ*

英語

Prop. 12

Let a cone be cut by a plane through the axis, and let it be cut by another plane cutting the base of the cone in

a straight line perpendicular to the base of the axial triangle, and let the diameter of the section, when produced, meet one side of the axial triangle beyond the vertex of the cone; then if any straight line be drawn from the section of the cone parallel to the common section of the cutting plane and the base of the cone as far as the diameter of the section, its square will be equal to the area applied to a certain straight line; this line is such that the straight line subtending the external angle of the triangle, lying in the same straight line with the diameter of the section, will bear to it the same ratio as the square on the line drawn from the vertex of the cone parallel to the diameter of the section as far as the base of the triangle bears to the rectangle bounded by the segments of the base made by the line so drawn; the breadth of the applied figure will be the intercept made by the ordinate on the diameter in the direction of the vertex of the section; and the applied figure will exceed by a figure similar and similarly situated to the rectangle bounded by the straight line subtending the external angle of the triangle and the parameter of the ordinates; and let such a section be called a hyperbola.

Let there be a cone whose vertex is the point Α and whose base is the circle ΒΓ, and let it be cut by a plane through the axis, and let the section so made be the triangle ΑΒΓ, and let it be cut by another plane cutting the base of the cone in the straight line ΔΕ perpendicular to ΒΓ, the base of the triangle ΑΒΓ, and let the section so made on the surface of the cone be the curve ΔΖΕ, and let ΖΗ, the diameter of the section, when produced, meet ΑΓ, one side of the triangle ΑΒΓ, beyond the vertex of the cone at Θ, and through Α let ΑΚ be drawn parallel to ΖΗ, the

diameter of the section, and let it cut ΒΓ, and from Ζ let ΖΛ be drawn perpendicular to ΖΗ, and let $ΚΑ^2 : ΒΚ . ΚΓ = ΖΘ : ΖΛ$, and let there be taken at random any point Μ on the section, and through Μ let ΜΝ be drawn parallel to ΔΕ, and through Ν let ΝΟΞ be drawn parallel to ΖΛ, and let ΘΛ be joined and produced to Ξ, and through Λ, Ξ, let ΛΟ, ΞΠ be drawn parallel to ΖΝ. I say that the square on ΜΝ is equal to ΖΞ, which is applied to the straight line ΖΛ, having ΖΝ for its breadth, and exceeding by the figure ΛΞ which is similar to the rectangle contained by ΘΖ, ΖΛ.

For let ΡΝΣ be drawn through Ν parallel to ΒΓ; but ΝΜ is parallel to ΔΕ; therefore the plane through ἄρα διὰ τῶν ΜΝ, ΡΣ ἐπίπεδον παράλληλόν ἐστι τῷ διὰ τῶν ΒΓ, ΔΕ, τουτέστι τῇ βάσει τοῦ κώνου. ἐὰν ἄρα ἐκβληθῇ τὸ διὰ τῶν ΜΝ, ΡΣ ἐπίπεδον, ἡ τομὴ κύκλος ἔσται, οὗ διάμετρος ἡ ΡΝΣ. καὶ ἔστιν ἐπ' αὐτὴν κάθετος ἡ ΜΝ· τὸ ἄρα ὑπὸ τῶν ΡΝΣ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΜΝ. καὶ ἐπεί ἐστιν, ὡς τὸ ἀπὸ ΑΚ πρὸς τὸ ὑπὸ ΒΚΓ, οὕτως ἡ ΖΘ πρὸς ΖΛ, ὁ δὲ τοῦ ἀπὸ τῆς ΑΚ πρὸς τὸ ὑπὸ ΒΚΓ λόγος σύγκειται ἔκ τε τοῦ, ὃν ἔχει ἡ ΑΚ πρὸς ΚΓ καὶ ἡ ΑΚ πρὸς ΚΒ, καὶ ὁ τῆς ΖΘ ἄρα πρὸς τὴν ΖΛ λόγος σύγκειται ἐκ τοῦ, ὃν ἔχει ἡ ΑΚ πρὸς ΚΓ καὶ ἡ ΑΚ πρὸς ΚΒ. ἀλλ' ὡς μὲν ἡ ΑΚ πρὸς ΚΓ, οὕτως ἡ ΘΗ πρὸς ΗΓ, τουτέστιν ἡ ΘΝ πρὸς ΝΣ, ὡς δὲ ἡ ΑΚ πρὸς ΚΒ, οὕτως ἡ ΖΗ πρὸς ΗΒ, τουτέστιν ἡ ΖΝ πρὸς ΝΡ· ὁ ἄρα τῆς ΘΖ πρὸς ΖΛ λόγος σύγκειται ἔκ τε τοῦ τῆς ΘΝ πρὸς ΝΣ καὶ τοῦ τῆς ΖΝ πρὸς ΝΡ. ὁ δὲ συγκείμενος λόγος ἐκ τοῦ τῆς ΘΝ πρὸς ΝΣ καὶ τοῦ τῆς ΖΝ πρὸς ΝΡ ὁ τοῦ ὑπὸ τῶν ΘΝΖ ἐστι πρὸς τὸ ὑπὸ τῶν ΣΝΡ· καὶ ὡς ἄρα τὸ ὑπὸ τῶν ΘΝΖ πρὸς τὸ ὑπὸ τῶν ΣΝΡ, οὕτως ἡ ΘΖ πρὸς ΖΛ, τουτέστιν ἡ ΘΝ πρὸς ΝΞ. ἀλλ' ὡς ἡ ΘΝ πρὸς ΝΞ, τῆς ΖΝ κοινοῦ ὕψους λαμβανομένης οὕτως τὸ ὑπὸ τῶν ΘΝΖ πρὸς τὸ ὑπὸ τῶν ΖΝΞ. καὶ ὡς ἄρα τὸ ὑπὸ τῶν ΘΝΖ πρὸς τὸ ὑπὸ τῶν ΣΝΡ, οὕτως τὸ ὑπὸ τῶν ΘΝΖ πρὸς τὸ ὑπὸ τῶν ΞΝΖ. τὸ ἄρα ὑπὸ ΣΝΡ ἴσον ἐστὶ τῷ ὑπὸ ΞΝΖ. τὸ δὲ ἀπὸ ΜΝ ἴσον ἐδείχθη τῷ ὑπὸ ΣΝΡ· καὶ τὸ ἀπὸ τῆς ΜΝ ἄρα ἴσον ἐστὶ τῷ ὑπὸ τῶν ΞΝΖ. τὸ δὲ ὑπὸ ΞΝΖ ἐστι τὸ ΞΖ παραλληλόγραμμον. ἡ ἄρα

ΜΝ δύναται τὸ ΞΖ, ὃ παράκειται παρὰ τὴν ΖΛ, πλάτος ἔχον τὴν ΖΝ, ὑπερβάλλον τῷ ΛΞ ὁμοίῳ ὄντι τῷ ὑπὸ τῶν ΘΖΛ. καλείσθω δὲ ἡ μὲν τοιαύτη τομὴ ὑπερβολή, ἡ δὲ ΛΖ παρ' ἣν δύνανται αἱ ἐπὶ τὴν ΖΗ καταγόμεναι τεταγμένως· καλείσθω δὲ ἡ αὐτὴ καὶ ὀρθία, πλαγία δὲ ἡ ΖΘ.

MN, PΣ is parallel to the plane through BΓ, ΔE [Eucl. xi. 15], that is to the base of the cone. If, then, the plane through MN, PΣ be produced, the section will be a circle with diameter PNΣ [Prop. 4]. And MN is perpendicular to it; therefore

$$PN \,.\, N\Sigma = MN^2.$$

And since $AK^2 : BK \,.\, K\Gamma = Z\Theta : Z\Lambda$,

while $AK^2 : BK \,.\, K\Gamma = (AK : K\Gamma)(AK : KB)$,

therefore $Z\Theta : Z\Lambda = (AK : K\Gamma)(AK : KB)$.

But $AK : K\Gamma = \Theta H : H\Gamma$,

i.e., $= \Theta N : N\Sigma$, [Eucl. vi. 4

and $AK : KB = ZH : HB$,

i.e., $= ZN : NP$. [*ibid.*

Therefore $\Theta Z : Z\Lambda = (\Theta N : N\Sigma)(ZN : NP)$.

But $(\Theta N : N\Sigma)(ZN : NP) = \Theta N \,.\, NZ : \Sigma N \,.\, NP$;

and therefore

$$\Theta N \,.\, NZ : \Sigma N \,.\, NP = \Theta Z : Z\Lambda$$

$$= \Theta N : N\Xi. \quad \text{[ibid.}$$

But $\Theta N : N\Xi = \Theta N \,.\, NZ : ZN \,.\, N\Xi$,

by taking a common height ZN.

And therefore

$$\Theta N \,.\, NZ : \Sigma N \,.\, NP = \Theta N \,.\, NZ : \Xi N \,.\, NZ.$$

Therefore $\Sigma N \,.\, NP = \Xi N \,.\, NZ$. [Eucl. v. 9

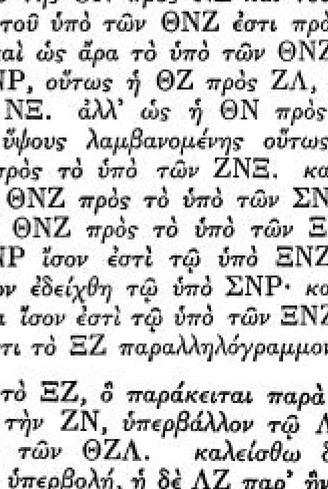
But $MN^2 = \Sigma N \,.\, NP$,

as was proved;

and therefore $MN^2 = \Xi N \,.\, NZ$.

But the rectangle $\Xi N \,.\, NZ$ is the parallelogram ΞZ.

Therefore the square on MN is equal to ΞZ, which is applied to ZΛ, having ZN for its breadth, and exceeding by ΛΞ similar to the rectangle contained by ΘZ, ZΛ. Let such a section be called a ***hyperbola***, let ΛZ be called ***the parameter to the ordinates*** to ZH; and let this line be also called the ***erect side*** (*latus rectum*), and ZΘ the ***transverse side***.[a]



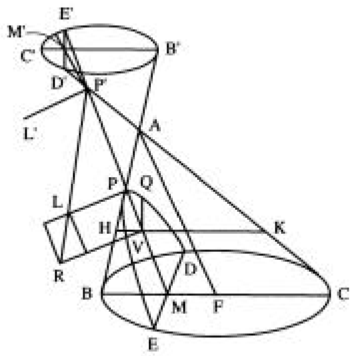
命題 12

頂点が点 A **で、底面が** BC **を直径とする円である円錐について考える。軸を通る面で切断し、できた切断面を三角形** ABC **とする。その三角形の底辺** BC **に垂直となる面で切断し、その切り口を** EPD **とする。そして、**PM **が** CA **の延長と** P’**で交わるようにする。また、点** A **を通り** PM **に平行な直線をひき、**BC **との交点を** F **とする。点** P **から切り口（**PM**）に垂直になるように** PL **を描き、その長さを**

$PL : PP' = BF \cdot FC : AF^2$

となるようにする。点 Q **をその切り口上に任意に取り、点** Q **を通り、**DE **に平行になるように** QV **をひく。**P’L **を結び** PL **に平行に** VR **をひき、**P’L **の延長との交点を** R **とする。**

このとき、$QV^2 = PV \cdot VR$　**であるといえる。**



$QV^2 = PV \cdot VR$ **とは・・・**

QV **を一辺とする正方形の面積に等しい長方形を** PL **上に、幅を** PV **となるようにつくると、これは** PV **と** PL **とを辺とする長方形よりも長方形** LR **の面積だけ超過する。**

そこでアポロニウスは、

hyperbola（ ὑπερβολή 、**超過する）、と名づけた。日本語では双曲線である。**

<u>作図の手順</u>

①

②

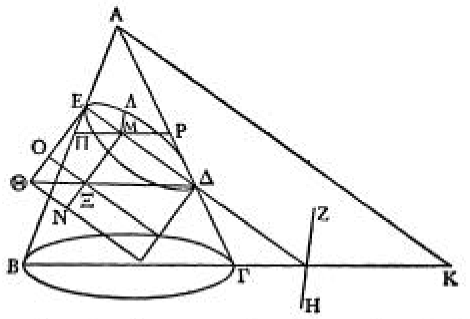
③

④

ιγ'

Ἐὰν κῶνος ἐπιπέδῳ τμηθῇ διὰ τοῦ ἄξονος, τμηθῇ δὲ καὶ ἑτέρῳ ἐπιπέδῳ συμπίπτοντι μὲν ἑκατέρᾳ πλευρᾷ τοῦ διὰ τοῦ ἄξονος τριγώνου, μήτε δὲ παρὰ τὴν βάσιν τοῦ κώνου ἠγμένῳ μήτε ὑπεναντίως, τὸ δὲ ἐπίπεδον, ἐν ᾧ ἐστιν ἡ βάσις τοῦ κώνου, καὶ τὸ τέμνον ἐπίπεδον συμπίπτῃ κατ' εὐθεῖαν πρὸς ὀρθὰς οὖσαν ἤτοι τῇ βάσει τοῦ διὰ τοῦ ἄξονος τριγώνου ἢ τῇ ἐπ' εὐθείας αὐτῇ, ἥτις ἂν ἀπὸ τῆς τομῆς τοῦ κώνου παράλληλος ἀχθῇ τῇ κοινῇ τομῇ τῶν ἐπιπέδων ἕως τῆς διαμέτρου τῆς τομῆς, δυνήσεταί τι χωρίον παρακείμενον παρά τινα εὐθεῖαν, πρὸς ἣν λόγον ἔχει ἡ διάμετρος τῆς τομῆς, ὃν τὸ τετράγωνον τὸ ἀπὸ τῆς ἠγμένης ἀπὸ τῆς κορυφῆς τοῦ κώνου παρὰ τὴν διάμετρον τῆς τομῆς ἕως τῆς βάσεως τοῦ τριγώνου πρὸς τὸ περιεχόμενον ὑπὸ τῶν ἀπολαμβανομένων ὑπ' αὐτῆς πρὸς ταῖς τοῦ τριγώνου εὐθείαις, πλάτος ἔχον τὴν ἀπολαμβανομένην ὑπ' αὐτῆς ἀπὸ τῆς διαμέτρου πρὸς τῇ κορυφῇ τῆς τομῆς, ἐλλεῖπον εἴδει ὁμοίῳ τε καὶ ὁμοίως κειμένῳ τῷ περιεχομένῳ ὑπό τε τῆς διαμέτρου καὶ τῆς παρ' ἣν δύνανται· καλείσθω δὲ ἡ τοιαύτη τομὴ ἔλλειψις.

Ἔστω κῶνος, οὗ κορυφὴ μὲν τὸ Α σημεῖον, βάσις δὲ ὁ ΒΓ κύκλος, καὶ τετμήσθω ἐπιπέδῳ διὰ τοῦ ἄξονος, καὶ ποιείτω τομὴν τὸ ΑΒΓ τρίγωνον, τετμήσθω δὲ καὶ ἑτέρῳ ἐπιπέδῳ συμπίπτοντι μὲν ἑκατέρᾳ πλευρᾷ τοῦ διὰ τοῦ ἄξονος τριγώνου, μήτε δὲ παραλλήλῳ τῇ βάσει τοῦ κώνου μήτε ὑπεναντίως ἠγμένῳ, καὶ ποιείτω τομὴν ἐν τῇ ἐπιφανείᾳ τοῦ κώνου τὴν ΔΕ γραμμήν· κοινὴ



δὲ τομὴ τοῦ τέμνοντος ἐπιπέδου καὶ τοῦ, ἐν ᾧ ἐστιν ἡ βάσις τοῦ κώνου, ἔστω ἡ ΖΗ πρὸς ὀρθὰς οὖσα τῇ ΒΓ, ἡ δὲ διάμετρος τῆς τομῆς ἔστω ἡ ΕΔ, καὶ ἀπὸ τοῦ Ε τῇ ΕΔ πρὸς ὀρθὰς ἤχθω ἡ ΕΘ, καὶ διὰ τοῦ Α τῇ ΕΔ παράλληλος ἤχθω ἡ ΑΚ, καὶ πεποιήσθω ὡς τὸ ἀπὸ ΑΚ πρὸς τὸ ὑπὸ ΒΚΓ, οὕτως ἡ ΔΕ πρὸς τὴν ΕΘ, καὶ εἰλήφθω τι σημεῖον ἐπὶ τῆς τομῆς τὸ Λ, καὶ διὰ τοῦ Λ τῇ ΖΗ παράλληλος ἤχθω ἡ ΛΜ. λέγω, ὅτι ἡ ΛΜ δύναταί τι χωρίον, ὃ παράκειται παρὰ τὴν ΕΘ, πλάτος ἔχον τὴν ΕΜ, ἐλλεῖπον εἴδει ὁμοίῳ τῷ ὑπὸ τῶν ΔΕΘ.

Ἐπεζεύχθω γὰρ ἡ ΔΘ, καὶ διὰ μὲν τοῦ Μ τῇ

Prop. 13

Let a cone be cut by a plane through the axis, and let it be cut by another plane meeting each side of the axial triangle, being neither parallel to the base nor subcontrary, and let the plane containing the base of the cone meet the cutting plane in a straight line perpendicular either to the base of the axial triangle or to the base produced; then if a straight line be drawn from any point of the section of the cone parallel to the common section of the planes as far as the diameter of the section, its square will be equal to an area applied to a certain straight line; this line is such that the diameter of the section will bear to it the same ratio as the square on the line drawn from the vertex of the cone parallel to the diameter of the section as far as the base of the triangle bears to the rectangle contained by the intercepts made by it on the sides of the triangle; the breadth of the applied figure will be the intercept made by it on the diameter in the direction of the vertex of the section; and the applied figure will be deficient by a figure similar and similarly situated to the rectangle bounded by the diameter and the parameter; and let such a section be called an ellipse.

Let there be a cone, whose vertex is the point A and whose base is the circle ΒΓ, and let it be cut by a plane through the axis, and let the section so made be the triangle ΑΒΓ, and let it be cut by another plane meeting either side of the axial triangle, being drawn neither parallel to the base nor subcontrary, and let the section made on the surface of the cone be the curve ΔΕ; let the common section of the cutting plane and of that containing the base of the cone be ΖΗ, perpendicular to ΒΓ, and let the diameter of the section be ΕΔ, and from Ε let ΕΘ be drawn perpendicular to ΕΔ, and through Α let ΑΚ be drawn parallel to ΕΔ, and let $AK^2 : BK \cdot K\Gamma = \Delta E : E\Theta$, and let any point Λ be taken on the section, and through Λ let ΛΜ be drawn parallel to ΖΗ. I say that the square on ΛΜ is equal to an area applied to the straight line ΕΘ, having ΕΜ for its breadth, and being deficient by a figure similar to the rectangle contained by ΔΕ, ΕΘ.

For let ΔΘ be joined, and through Μ let ΜΞΝ be

ΘΕ παράλληλος ἤχθω ἡ ΜΞΝ, διὰ δὲ τῶν Θ, Ξ τῇ ΕΜ παράλληλοι ἤχθωσαν αἱ ΘΝ, ΞΟ, καὶ διὰ τοῦ Μ τῇ ΒΓ παράλληλος ἤχθω ἡ ΠΜΡ. ἐπεὶ οὖν ἡ ΠΡ τῇ ΒΓ παράλληλός ἐστιν, ἔστι δὲ καὶ ἡ ΛΜ τῇ ΖΗ παράλληλος, τὸ ἄρα διὰ τῶν ΛΜ, ΠΡ ἐπίπεδον παράλληλόν ἐστι τῷ διὰ τῶν ΖΗ, ΒΓ ἐπιπέδῳ, τουτέστι τῇ βάσει τοῦ κώνου. ἐὰν ἄρα ἐκβληθῇ διὰ τῶν ΛΜ, ΠΡ ἐπίπεδον, ἡ τομὴ κύκλος ἔσται, οὗ διάμετρος ἡ ΠΡ. καί ἐστι κάθετος ἐπ' αὐτὴν ἡ ΛΜ· τὸ ἄρα ὑπὸ τῶν ΠΜΡ ἴσον ἐστὶ τῷ ἀπὸ τῆς ΛΜ. καὶ ἐπεί ἐστιν, ὡς τὸ ἀπὸ τῆς ΑΚ πρὸς τὸ ὑπὸ τῶν ΒΚΓ, οὕτως ἡ ΕΔ πρὸς τὴν ΕΘ, ὁ δὲ τοῦ ἀπὸ τῆς ΑΚ πρὸς τὸ ὑπὸ τῶν ΒΚΓ λόγος σύγκειται ἐκ τοῦ, ὃν ἔχει ἡ ΑΚ πρὸς ΚΒ, καὶ ἡ ΑΚ πρὸς ΚΓ, ἀλλ' ὡς μὲν ἡ ΑΚ πρὸς ΚΒ, οὕτως ἡ ΕΗ πρὸς ΗΒ, τουτέστιν ἡ ΕΜ πρὸς ΜΠ, ὡς δὲ ἡ ΑΚ πρὸς ΚΓ, οὕτως ἡ ΔΗ πρὸς ΗΓ, τουτέστιν ἡ ΔΜ πρὸς ΜΡ, ὁ ἄρα τῆς ΔΕ πρὸς τὴν ΕΘ λόγος σύγκειται ἔκ τε τοῦ τῆς ΕΜ πρὸς ΜΠ καὶ τοῦ τῆς ΔΜ πρὸς ΜΡ. ὁ δὲ συγκείμενος λόγος ἔκ τε τοῦ, ὃν ἔχει ἡ ΕΜ πρὸς ΜΠ, καὶ ἡ ΔΜ πρὸς ΜΡ, ὁ τοῦ ὑπὸ τῶν ΕΜΔ ἐστι πρὸς τὸ ὑπὸ τῶν ΠΜΡ. ἔστιν ἄρα ὡς τὸ ὑπὸ τῶν ΕΜΔ πρὸς τὸ ὑπὸ τῶν ΠΜΡ, οὕτως ἡ ΔΕ πρὸς τὴν ΕΘ, τουτέστιν ἡ ΔΜ πρὸς τὴν ΜΞ. ὡς δὲ ἡ ΔΜ πρὸς ΜΞ, τῆς ΜΕ κοινοῦ ὕψους λαμβανομένης, οὕτως τὸ ὑπὸ ΔΜΕ πρὸς τὸ ὑπὸ ΞΜΕ. καὶ ὡς ἄρα τὸ ὑπὸ ΔΜΕ πρὸς τὸ ὑπὸ ΠΜΡ, οὕτως τὸ ὑπὸ ΔΜΕ πρὸς τὸ ὑπὸ ΞΜΕ. ἴσον ἄρα ἐστὶ τὸ ὑπὸ ΠΜΡ τῷ ὑπὸ ΞΜΕ. τὸ δὲ ὑπὸ ΠΜΡ ἴσον ἐδείχθη τῷ ἀπὸ τῆς ΛΜ· καὶ τὸ ὑπὸ ΞΜΕ ἄρα ἐστὶν ἴσον τῷ ἀπὸ τῆς ΛΜ. ἡ ΛΜ ἄρα δύναται τὸ ΜΟ, ὃ παράκειται παρὰ τὴν ΘΕ, πλάτος ἔχον τὴν ΕΜ, ἐλλεῖπον εἴδει τῷ ΟΝ ὁμοίῳ ὄντι τῷ ὑπὸ ΔΕΘ. καλείσθω δὲ ἡ μὲν τοιαύτη τομὴ ἔλλειψις, ἡ δὲ ΕΘ παρ' ἣν δύνανται αἱ καταγόμεναι ἐπὶ τὴν ΔΕ τεταγμένως, ἡ δὲ αὐτὴ καὶ ὀρθία, πλαγία δὲ ἡ ΕΔ.

drawn parallel to ΘΕ, and through Θ, Ξ, let ΘΝ, ΞΟ be drawn parallel to ΕΜ, and through Μ let ΠΜΡ be drawn parallel to ΒΓ. Then since ΠΡ is parallel to ΒΓ, and ΛΜ is parallel to ΖΗ, therefore the plane through ΛΜ, ΠΡ is parallel to the plane through ΖΗ, ΒΓ [Eucl. xi. 15], that is to the base of the cone. If, therefore, the plane through ΛΜ, ΠΡ be produced, the section will be a circle with diameter ΠΡ [Prop. 4]. And ΛΜ is perpendicular to it ; therefore

$$\Pi M . MP = \Lambda M^2.$$

And since $AK^2 : BK . K\Gamma = E\Delta : E\Theta$,

and $AK^2 : BK . K\Gamma = (AK : KB)(AK : K\Gamma)$,

while $AK : KB = EH : HB$

$= EM : M\Pi$, [Eucl. vi. 4

and $AK : K\Gamma = \Delta H : H\Gamma$

$= \Delta M : MP$, [*ibid.*

therefore $\Delta E : E\Theta = (EM : M\Pi)(\Delta M : MP)$.

But $(EM : M\Pi)(\Delta M : MP) = EM . M\Delta : \Pi M . MP$.

Therefore

$$EM . M\Delta : \Pi M . MP = \Delta E : E\Theta$$

$= \Delta M : M\Xi$. [*ibid.*

But $\Delta M : M\Xi = \Delta M . ME : \Xi M . ME$,

by taking a common height ME.

Therefore $\Delta M . ME : \Pi M . MP = \Delta M . ME : \Xi M . ME$.

Therefore $\Pi M . MP = \Xi M . ME$. [Eucl. v. 9

But $\Pi M . MP = \Lambda M^2$,

as was proved ;

and therefore $\Xi M . ME = \Lambda M^2$.

Therefore the square on ΛΜ is equal to ΜΟ, which is applied to ΘΕ, having ΕΜ for its breadth, and being deficient by the figure ΟΝ similar to the rectangle ΔΕ . ΕΘ. Let such a section be called an *ellipse*, let ΕΘ be called *the parameter to the ordinates* to ΔΕ, and let this line be called the *erect side* (*latus rectum*), and ΕΔ the *transverse side*.[a]

命題 13

頂点が点 A で、底面が BC を直径とする円である円錐について考える。軸を通る面で切断し、できた切断面を三角形 ABC とする。その三角形の底辺に平行ではなく、両方の母線に出会う面で切断し、その切り口を PP'とし、PM が AC と P'で交わるようにする。また、点 A を通り PM に平行な直線をひき、BC の延長線との交点を F とする。点 P から切り口（PP'）に垂直になるように PL を描き、その長さを

PL：PP'＝BF・FC：AF^2

となるようにする。点 Q をその切り口上に任意に取り、点 Q を通り、DE に平行になるように QV をひく。P'L を結び PL に平行に VR をひき、P'L との交点を R とする。

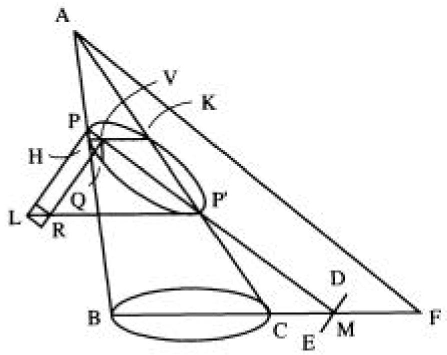
このとき、**QV^2＝PV・VR**　であるといえる。

QV^2＝PV・VR とは・・・

QV を一辺とする正方形の面積に等しい長方形を PL 上に、幅を PV となるようにつくると、これは PV と PL とを辺とする長方形よりも長方形 LR の面積だけ不足する。

そこでアポロニウスは、

ellipse（ἔλλειψις　、不足する)、と名づけた。日本語では楕円である。



<u>作図の手順</u>

①

②

③

④

まとめ

次の場合、切断面の切り口はどのような円錐曲線を描きますか？

（I ）　円錐の母線と切断面のなす角が頂角に等しいとき（母線と切断面が平行になるとき）

____

（II）　円錐の母線と切断面のなす角が頂角よりも小さいとき

____

（III）　円錐の母線と切断面のなす角が頂角よりも大きいとき

____

問　北半球が夏至のときを考えてみましょう。

地面に垂直にグノモンを立てたとき、その影の軌跡が次の円錐曲線となる地域はどこでしょうか。

楕円となるのは

____

双曲線となるのは

____

放物線となるのは

____

斜円錐の切断面（3 種類の円錐曲線）

命題 11　　**命題** 12　　**命題** 13

$QV^2 = PL \cdot PV$　　**$QV^2 = PV \cdot VR$**　　**$QV^2 = PV \cdot VR$**