TEACHING EXPERIMENT OF MATHEMATICS WITH GRAPES IN BOSNIA AND HERZEGOVINA

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1. Introduction

At the CRICED[1] of the University of Tsukuba, in Japan, we are now conducting a project named "*CRICED - JICA Joint Project*" in the domain of International Cooperation on Education. This project is financed by the Ministry of Education in Japan from April 2005 to March 2010 in order to promote research cooperation between the University and institutions from different sectors. Japan International Cooperation Agency (JICA) is the main partner of our project.

CRICED has been working so far with JICA in the framework of JICA project for educational development. The CRICED staff often participates in JICA projects as short-term experts and hosts and organizes short-term or long-term JICA training programmes. The training programme "*Promotion of Information and Communication Technology (ICT) Education, and Development for e-learning in Informatics and Mathematics at Elementary and Secondary Levels for Bosnia and Herzegovina*" (representative: Masami ISODA) is one of JICA projects hosted at CRICED, University of Tsukuba. In this JICA training programme, the role of CRICED is to host the trainees. Therefore, we were looking for means to enhance cooperation with Bosnian and Herzegovinian people even after the programme. Thus, the project "*CRICED – JICA Joint Project*" has been started with the goals of enhancing the projects conducted by JICA and promoting educational development not only during the training programme but also after.

In July 2005, as an activity of this project, we organized a meeting and a seminar in Tsukuba with Bosnian and Herzegovinian professors from Sarajevo, Mostar and Banja Luka. The topic of discussion was the cooperative activities in the domain of mathematics and informatics education with ICT (Information and Communication Technology) in the framework of CRICED – JICA joint project. At that moment, we decided to use the free software GRAPES in order to get started on our activities.

On this background, a work visit to Bosnia and Herzegovina took place and there a

[1] Center for Research on International Cooperation in Educational Development

mathematics teaching experiment with GRAPES was organized. This was a good moment in the sense that first JICA trainees of the year 2004/2005 came back to their country and we are able to work together. In this paper, we will first describe the features of GRAPES and then report a teaching experiment with GRAPES conducted in Prijedor. For the other experiments done in Bosnia and Herzegovina, we will detail them in other places.

The experiment was realized with the cooperation of Mr. Ljubomir Petkovic, professor at the Machine Technical High School in Prijedor, who was the previous JICA trainee 2004/2005 in CRICED – University of Tsukuba and Ms. Dragana Jejetovic, professor of mathematics and computer science at Gimnazija 'Sveti Sava'.

2. Feature of GRAPES

GRAPES is a graphing software that allows graphical representations of most of the functions and relations which appear at the secondary and undergraduate levels. It has almost all the functionalities of a graphing calculator except numerical and algebraic calculations. GRAPES is not a CAS (Computer Algebra System) like Mathematica or Drive in the sense that it has been developed just for the educational purpose of visualizing functions and relations especially for secondary mathematics.

The first function as importance of GRAPES is, as for other graphing software, the drawing of graphs for a given algebraic expression. This function has the potential to dramatically change the teaching and learning of mathematical functions (see Romberg et al (eds.), 1993)[2]. Traditionally, only few specific types of functions having specific symbolic expressions such as the linear function or the quadratic function are easily drawn on paper. Graphing software enables the immediate drawing of graphs and allows students not only to explore several traditional graphs of functions but also starting from these to derive graphs of complicated algebraic expressions and thus to understand their classification within the traditional types. Moreover, whereas, in the traditional way, the graph was often the ending point of mathematical problems, now the approach from the graphical representation, such as finding the algebraic expression to a given graph or given phenomena, is also possible.

The second essential function of GRAPES refers to parameters and locus. Using parameters and locus, it's easy to visualize families of curves of functions and relations. The roles of parameters in the function are visible through exploring the graphical and analytical nature of certain families (see Figure 1, showing the dynamic of the focal point in a family of parabolas when the b coefficient of the quadratic function $y = ax^2 + bx + c$ varies). The fact that parametric curves can be easily drawn is another quality of the GRAPES worth mentioning.

[2] Romberg, T.A., Fennema, E. and Carpenter, T.P. (Eds.). (1993). *Integrating Research on the Graphical Representations of Functions.* Hillsdale, NJ: Erlbaum.

Graphing calculators are actually not common in Japanese classrooms. GRAPES is one of the most used software in ordinary mathematics classes in Japan for two main reasons. Firstly its ease of use, the highly friendly interface makes it easy to employ the software in the limited time of a lesson. The second reason is its low cost as a *Freeware*.

GRAPES was developed by a high school teacher Katsuhisa TOMODA (Ikeda Senior High School within the Osaka Kyoiku University). He has not developed it for commercial purposes, but just for his and his colleagues use in order to enrich and make their mathematics classes more active. This kind of freeware for educational use is often developed in Japan. For example, a dynamic geometry freeware for geometry learning, the Geometric Constructor can also be easily downloaded. GRAPES has been translated in English and in Spanish and is downloadable from the: http://www.criced.tsukuba.ac.jp/grapes/ website.

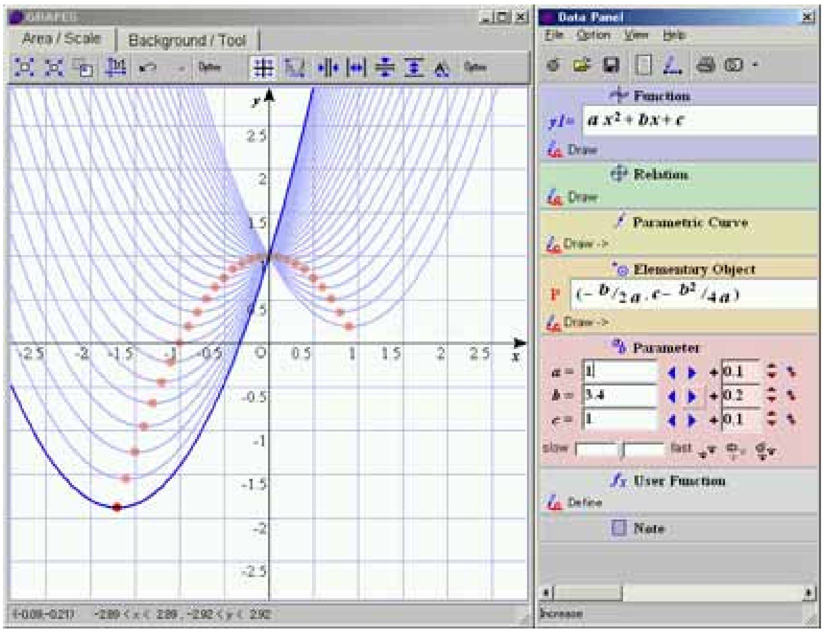


Figure 1. Interface of GRAPES
http://www.criced.tsukuba.ac.jp/grapes/

3. Teaching experiment with GRAPES

The teaching experiment was organized at the secondary school in Prijedor, Gimnazija 'Sveti Sava'. This is an old secondary school which was established in 1921 and situated in the town center (see Figure 2). The main objective of this teaching experiment was, as this was our first attempt to use this software in Bosnia and Herzegovina, *to see the adaptability and the potential of the Japanese free software GRAPES translated into English in a class in Bosnia and Herzegovina*. The "adaptability" referred to here has several points to consider: infrastructure, educational system, curriculum, teachers skill, motivation, etc.

For the experiment, Mr. Ljubomir Petkovic asked Ms. Dragana Jejetovic, professor of mathematics and informatics at the Gimnazija ‘Sveti Sava’, to organize a class using GRAPES at the secondary school in Prijedor. Before going there, we had sent some information and examples for the use of GRAPES in mathematics teaching. The class itself, 3rd year mathematics of secondary school[3], was planed by the teacher. The class of students under observation had already been introduced to the use of GRAPES. In what follows a global view of this experimental class is given. Details, the plan and worksheets for the class, can be found in the appendix to this paper.

Figure 2 Gimnazija ‘Sveti Sava’

Grade: 3rd year of secondary school at Prijedor (Gimnazija ‘Sveti Sava’)

Teacher: Ms. Dragana Jejetovic

Domain: Graphs of function

Date: 1st December 2005 from 7:30 – 9:00 (two consecutive classes)

Objectives[4]:

1. Reviewing and consolidating the knowledge of some relationships between the graphical and algebraic representation of a function ($y = f(x)$, $y_1 = f(x) + m$, $y_1 = f(x+n)$, $y_1 = -f(x)$, $y_1 = |f(x)|$) and of some analytic properties of a function (the sign of the function, where is it decreasing and increasing, etc.);
2. Applying the consolidated knowledge in solving problems of min and max volume determination.

Materials: Computers for students (one per pair), GRAPES software, worksheets

3.1 Analysis of exercises

Exercise 1 (appendix p.2) is given for the reviewing of previous knowledge on the relationship between graphical representation and algebraic representation:. The exercise consisted in observing through GRAPES the graphs of related functions such as $y_1 = f(x) + m$, $y_1 = f(x+n)$, $y_1 = -f(x)$, $y_1 = |f(x)|$ for the given real value function $f(x) = \log_2 x$. Here is given just one example.

$y_1 = f(x) + m$

The graph of the function $y_1 = f(x) + m$ is created by ________ of the graph of the

[3] The educational system in Bosnia and Herzegovina is 8 years of primary school and 4 years of secondary school. The 3rd year of secondary school in Bosnia and Herzegovina corresponds to the 2nd year of higher secondary school in Japan. The Gimnazija is a general secondary school for students and usually gathers good students.

[4] These objectives are cited from the plan written by the teacher.

function y = f(x) in the direction of ___-axis for _______

For m>0 a) up b) down (circle the correct answer)

For m<0 a) up b) down (circle the correct answer)

As this is an exercise whose objective is to review the already acquired knowledge, the role of "m" in this function had been more or less known by the students. What GRAPES allows in this exercise is to draw graphs of the function immediately and to show how the graph changes according to the value of *m*. Usually, the task of drawing graphs, as that for $y_1 = f(x) + m$ cannot be accomplished immediately, it requires a certain amount of time. The drawing of a graph of $f(x) = \log_2 x$ requires a number of steps: inserting a value for *x* in the expression, calculating, plotting on the plan, and repeating the same sequence of steps. Instead, GRAPES allows to **save time** at this point and to **<u>explore</u>** the graph of the function.

This economy of time is more obvious out of exercise 3 (see appendix pp.3-4). The problem requires the students to solve an inequality which is quite a complex expression. The inequality given is $|\log_2(x + 2)|(-2^x + 2)(\sqrt[3]{x} + 1) > 0$. The approach to handling the problem envisaged by the teacher was an algebraic one. The teacher expected to solve this by using the results of exercise 2 and sign tables for separated domains: look for the sign of the function by looking at the sign of each part of the function, which were for convenience treated as new functions: ($f_1(x) = |\log_2(x + 2)|$, $f_2(x) = (-2^x + 2)$, $f_3(x) = (\sqrt[3]{x} + 1)$). After having solved the inequality, the task proposed was to check the result with GRAPES. The graph of the function for $f(x) = |\log_2(x + 2)|(-2^x + 2)(\sqrt[3]{x} + 1)$ would be almost impossible to draw for a 3rd grade student. After solving the inequality, GRAPES is used to check the solution the students got through the algebraic approach. In this case, GRAPES gives **an immediate feedback** which is not coming from the teacher but from the class environment. This feedback is possible to have owing to the graph's immediate drawing.

Exercises 4, 6, 7 and 8 are problems which usually require differentiation in order to get the local extrema of a function. By the 3rd grade, students have not yet acquire the knowledge of how to use this mathematical tool. This means that these exercises usually are not easy to tackle by students in this grade. However, as the GRAPES allows to automatically draw the graph of a function, which may be difficult to draw by hand, through **a quantitative approach or perceptive approach** it is possible to solve these problems. That is, the students can easily find the approximate values of the function at its minimum and maximum values. The students only have to put the cursor on the extreme point of the function graph and the value of the function at that point will appear on the bottom of the GRAPES window. This resolution process when students find approximate values doesn't really require mathematical knowledge. However, there are two outcomes which are rather important from the educational point of view. First, as we have already mentioned, GRAPES can be used to introduce beforehand problems that are difficult to be dealt with in class. Second, as the solution found by the perceptive approach is an approximate value, this makes the students to go to the next step with a clear problematic: how can one get an exact value of the extreme values for a function? Then, the qualitative or algebraic approach such as differentiation will be able to have a specific meaning for the students. In other words, this perceptive experience can be a motivation for employing the algebraic approach.

3.2 Observations

In what follows, we will comment on the observations made on the adaptability of GRAPES to the mathematics class in Bosnia and Herzegovina.

Infrastructure:

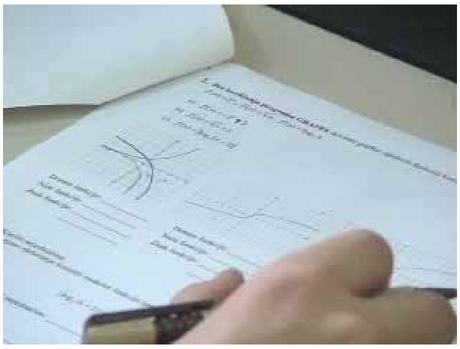
The number of computers was enough for the students in one class. As we mention below, this is due to the limited number of students per class. The desktop computers in the computer room are all Pentium II, equipped with the OS (Operating System) Windows 98. This configuration was enough for the use of GRAPES. Throughout the experimentation, no serious problems have occurred, the GRAPES working without at optimal speed. This is due to the small requirement of working memory the GRAPES package needs, less than 500 KB.

However, in the curriculum of the Gimnazija, the computer science class is given every year, from first year to the fourth year. As the students of this class usually use the computer room and of course they have priority, it is difficult to have the computer room available for the mathematics class. This constitutes an obstacle for further use of GRAPES in mathematics classes.

Number of students:

The number of students is limited to 25, by law. This number is much more pertinent for a class using computers, especially for individual or pair tasks using a computer and GRAPES. In the case of Japan, as the number of students in one class is 40, the teacher often cannot take care of all students. In particular, the use of a computer may cause several problems which may not be linked to the software, but to the hardware. This is why teachers in Japan mostly use GRAPES for exemplifications in front of the class.

Teacher's skill:

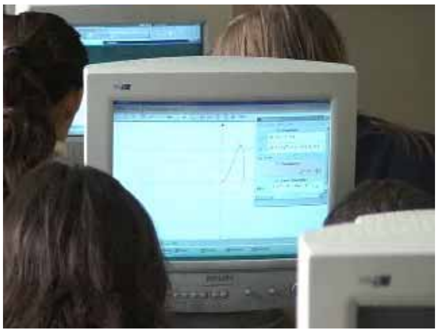


The teacher giving a class for our observation organized the students' tasks well, so as to bring out the potential of GRAPES. The software has been used as a time saving tool in order to underline the importance of the exploration by students, as a tool to give feedback to the students, and as a tool to tackle difficult problems through the quantitative approach and in consequence to attribute meaning to the algebraic approach. However, this might be due to the fact that the teacher in this experiment is a mathematics and computer science, and used to use computers in and out of class.

She told us that most mathematics teachers are not used to use computers even out of class. This is the same phenomenon as in Japan and also in other countries. The use of computers depends on the teachers' skills: the teachers who know well how to use computers will often organize a class using these tools, but the teachers who don't know will hesitate or do not want to use computers in the class. The teacher training for the use of computers would be indispensable.

English version of GRAPES:

For the present, the GRAPES is distributed in English, in Spanish and in Japanese. The official languages in Bosnia and Herzegovina are Serbian, Croatian and Bosnian. Therefore, we were thinking that software in the English language might be an obstacle for the students. However, we found that the students manipulated GRAPES without problem and it seems that English was not an obstacle for them, whereas it's sure that in Japan this would make a big obstacle.



4. Conclusion

The teaching experiment we have considered was well organized by the teacher Ms. Dragana Jejetovic. Our observation was done on a class of the students who had already learned the use of GRAPES. The first contact with GRAPES could not be observed this time. But we may suppose from students' manipulation of GRAPES, sometimes in an advanced way (definition of a function), that they did not have many difficulties in their first contact. We have seen that the software GRAPES was well adapted to a mathematics class in Bosnia and Herzegovina.

The JICA training programme for the use of ICT at the University of Tsukuba is organized in Japan. Our visit with the teaching experimentation location was the first tentative of effective cooperation on mathematics education in Bosnia and Herzegovina. We hope that this kind of effective activities will continue and be enhanced between the two countries.

Acknowledgements

The authors thank the Gimnazija ‘Sveti Sava’ for the organization of the teaching experiment in Bosnia and Herzegovina and in particular to the professor Dragana Jejetovic who accepted this experiment with a positive attitude. And of course, the visit to Prijedor was realized thanks to Mr. Ljubomir Petkovic, professor of Machine Technical High School in Prijedor, who was the previous JICA trainee 2004/2005 in CRICED – University of Tsukuba

Teaching experiment of mathematics with GRAPES

Grade: The third grade of the high school (Gimnazija 'Sveti Sava') in Prijedor

Domain: Graphs of function

Objectives:

1. Reviewing, revising and consolidating the knowledge of some relationships between graphical representation and algebraic representation of a function ($y=f(x)$, $y_1 = f(x)+m$, $y_1 = f(x+n)$, $y_1 = -f(x)$, $y_1 = |f(x)|$) and of some analytic properties of a function (the sign of the function, where it is decreasing and increasing, etc.)
2. Applying the consolidated knowledge to solving problems with min and max in stereometry

Note

The third grade students have not learned finding out a derivative of a function and using differentation to find max and min points of the graph of a function yet.

Now, as predicted by the annual plan and programme, they are learning stereometry.

GRAPES is an excelent software, that can help students to get information about the analytic properties of a function, such as approximate values of the coordinates of min and max points of the graph.

At present, the students can find out the solution of problems with a minimum and maximum in stereometry by observing the graph of the function.

The students learned some elementary functions, such as y=ax+bx+c, $y=a^x$, $y=\log_a x$, y=sinx, etc, in the second grade of the school. The students observed graphs of these functions, explored their properties, analysed the relationship betwen algebraic representation and graphical representation of these functions and analysed some analytic properties using the graphs of these functions (the sign of the function, where it is decreasing and increasing, etc.)

The purpose of the exercises (from 1 to 5) is to remind, review, revise, apply and consolidate the previously acquired knowledge about the following:

- the graphes of some of these functions
- some relationships between algebraic representation and graphical representation of the function
- exploring and finding out the sign of the function and some other analytic properties of the function by using the graph

Also, the purpose of all exercises is to learn and get a better insight into GRAPES and motivate the students for its further use

The students have the minimal skill for using the GRAPES.

The exercises are made by

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WORKSHEET

Date: **Grade:** **Name:**

Exercise 1. (Review)

The real valued function *f* is given:

$$f(x) = \log_2 x$$

Use GRAPES to draw graphs of the following functions for diferent values of the real parameters ***m*** and ***n***.
Observe the graphs of those functions.

a) $y_1 = f(x) + m$

The graph of the function $y_1 = f(x) + m$ is created by__________________of the graph of the function *y=f(x)* in the direction of______-axis for ______

- for **m>0** a) up b) down (circle the correct answer)
- for **m<0** a) up b) down (circle the correct answer)

b) $y_1 = f(x + n)$

The graph of the function $y_1 = f(x + n)$ is created by __________________of the graph of the function *y=f(x)* in the direction of______-axis for ______

- for **n>0** a) to the right b) to the left (circle the correct answer)
- for **n<0** a) to the right b) to the left (circle the correct answer)

c) $y_1 = -f(x)$

The graph of the function $y_1 = -f(x)$ is ________________with the graph of the function $y_1 = -f(x)$ to the_______-axis.

d) $y_1 = |f(x)|$

The graph of the function $y_1 = |f(x)|$ _______________ the graph of the function *y=f(x)* for ***f(x)≥0***, and is _____________________ to the_______-axis with the graph of the function *y=f(x)* for ***f(x)<0***.

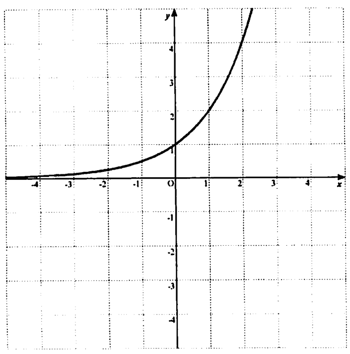
Exercise 2. (Consolidate)

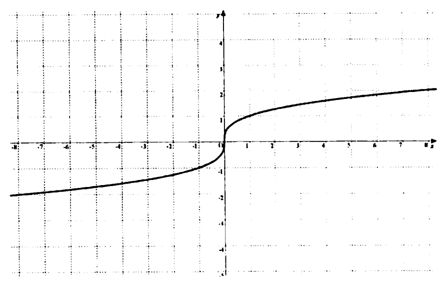
Draw the graphs of the following functions **without using GRAPES**. Use the graphs of drawn functions: $f(x)=2^x$, $f(x)=\sqrt[3]{x}$, $f(x)=\log_2 x$.

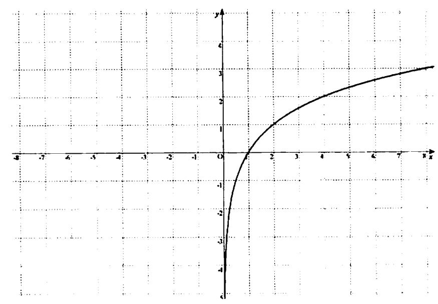
a) $f(x)=-2^x+2$

b) $f(x)=\sqrt[3]{x}+1$

c) $f(x)=|\log_2(x+2)|$







Domain: ____________
'Zero': ____________
The sign of function: ____________
____

Domain: ____________
'Zero': ____________
The sign of function: ____________
____

Domain: ____________
'Zero': ____________
The sign of function: ____________
____

Exercise 3. (Apply)

Solve the inequality.
Use the signs of the functions for solving the inequality (the previous exercise).

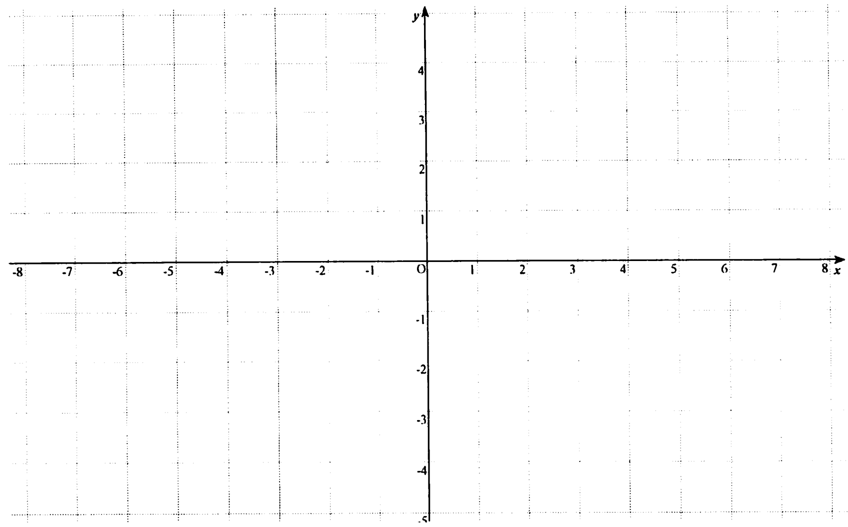
$$|\log_2(x+2)|(-2^x+2)(\sqrt[3]{x}+1)>0$$

The domain of expression in the inequality : ______________________________

x			
$\lvert\log(x+2)\rvert$			
-2^x+2			
$\sqrt[3]{x}+1$			
$\lvert\log_2(x+2)\rvert(-2^x+2)(\sqrt[3]{x}+1)$			

The solution interval of the inequality : ______________________________

Draw the graph of the function $f(x)=|\log_2(x+2)|(-2^x+2)(\sqrt[3]{x}+1)$ **by using GRAPES. Check the solution** of the inequality examining the sign of this function.



Exercise 4. (Rewiev)

By using the graph of the function $f(x)=\sqrt[3]{x^3+3x^2}$, exam the properties of the function *f*:

a) **the domain:** ______________________________

b) **the function is equals zero for :** ______________________________

c) **the sign of the function:**

  a. *f(x)*>0 for ____________

  b. *f(x)*<0 for ____________

d)

  a. **the function is decreasing in**______________________

  b. **the function is increasing in**______________________

e) **The function has** ________________ **relative extremums**

(the number of)

f) **the relative extremums are:** ______________________________

____

g) **The function is** (circle the correct answer)

  a. 'pair'

  b. 'add'

  c. neither

Explain!

Exercise 6. (Apply)

Find the approximate values of the dimensions (the base radius and the height) of the cone with the largest (maximal) volume for the given "slant height" , s=2cm.

V(r)=______________

For **r** ≈ ___________ the cone, with the given "slant height", s=2cm, has the largest (maximal) volume $\mathbf{V_{max}} \approx$ ____________

Exercise 7.

Find the approximate values of the dimensions of the cylinder with the smallest surface area for the given volume, V=2 m^3.

A(r)= ______________

For **r** ≈ ___________ and **H** ≈ ___________ the cylinder, with the given volume, V=2 $m^{\overset{3}{\cancel{2}}}$, has the smallest surface area $\mathbf{A_{min}} \approx$ ____________.

Exercise 8. (Homework)

Te cylinder with the largest area of the curved side surface (lateral area) is contained within the cone of height, h=2, and base radius, r=5. Find the approximate values of the dimensions of the cylinder.