

Teaching experiment of mathematics with GRAPES

Grade: The third grade of the high school (Gimnazija 'Sveti Sava') in Prijedor

Domain: Graphs of function

Objectives:

1. Reviewing, revising and consolidating the knowledge of some relationships between graphical representation and algebraic representation of a function ($y=f(x)$, $y_1 = f(x) + m$, $y_1 = f(x + n)$, $y_1 = -f(x)$, $y_1 = |f(x)|$) and of some analytic properties of a function (the sign of the function, where it is decreasing and increasing, etc.)
2. Applying the consolidated knowledge to solving problems with min and max in stereometry

Note

The third grade students have not learned finding out a derivative of a function and using differentiation to find max and min points of the graph of a function yet.

Now, as predicted by the annual plan and programme, they are learning stereometry.

GRAPES is an excellent software, that can help students to get information about the analytic properties of a function, such as approximate values of the coordinates of min and max points of the graph.

At present, the students can find out the solution of problems with a minimum and maximum in stereometry by observing the graph of the function.

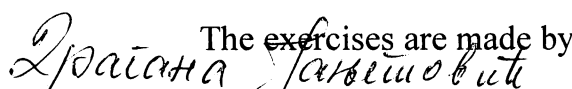
The students learned some elementary functions, such as $y=ax+bx+c$, $y=a^x$, $y=\log_a x$, $y=\sin x$, etc, in the second grade of the school. The students observed graphs of these functions, explored their properties, analysed the relationship between algebraic representation and graphical representation of these functions and analysed some analytic properties using the graphs of these functions (the sign of the function, where it is decreasing and increasing, etc.)

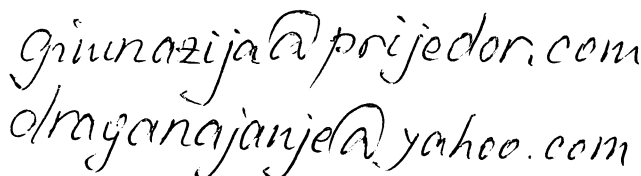
The purpose of the exercises (from 1 to 5) is to remind, review, revise, apply and consolidate the previously acquired knowledge about the following:

- the graphs of some of these functions
- some relationships between algebraic representation and graphical representation of the function
- exploring and finding out the sign of the function and some other analytic properties of the function by using the graph

Also, the purpose of all exercises is to learn and get a better insight into GRAPES and motivate the students for its further use

The students have the minimal skill for using the GRAPES.

The exercises are made by

Dragana Janjetic, professor of mathematics and computer science



WORKSHEET

Date: _____

Grade: _____

Name: _____

Exercise 1. (Review)

The real valued function f is given:

$$f(x) = \log_2 x$$

Use GRAPES to draw graphs of the following functions for different values of the real parameters m and n .

Observe the graphs of those functions.

a) $y_1 = f(x) + m$

The graph of the function $y_1 = f(x) + m$ is created by _____ of the graph of the function $y=f(x)$ in the direction of _____-axis for _____

- for $m > 0$ a) up b) down (circle the correct answer)
- for $m < 0$ a) up b) down (circle the correct answer)

b) $y_1 = f(x + n)$

The graph of the function $y_1 = f(x + n)$ is created by _____ of the graph of the function $y=f(x)$ in the direction of _____-axis for _____

- for $n > 0$ a) to the right b) to the left (circle the correct answer)
- for $n < 0$ a) to the right b) to the left (circle the correct answer)

c) $y_1 = -f(x)$

The graph of the function $y_1 = -f(x)$ is _____ with the graph of the function $y_1 = -f(x)$ to the _____-axis.

d) $y_1 = |f(x)|$

The graph of the function $y_1 = |f(x)|$ _____ the graph of the function $y=f(x)$ for $f(x) \geq 0$, and is _____ to the _____-axis with the graph of the function $y=f(x)$ for $f(x) < 0$.

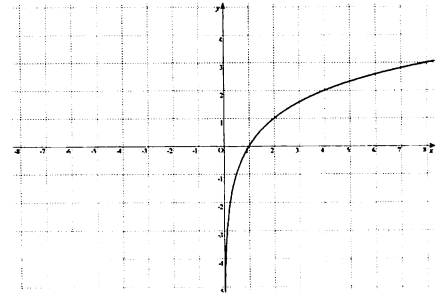
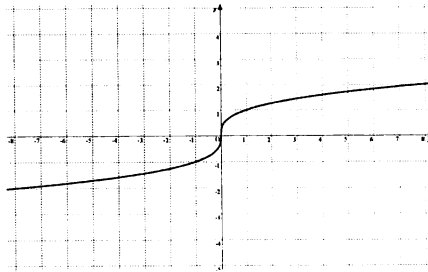
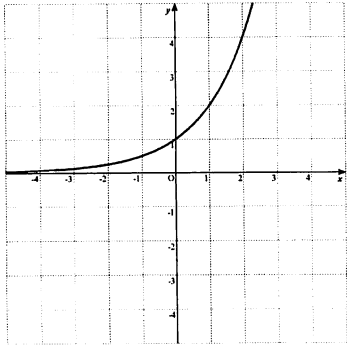
Exercise 2. (Consolidate)

Draw the graphs of the following functions **without using GRAPES**. Use the graphs of drawn functions: $f(x) = 2^x$, $f(x) = \sqrt[3]{x}$, $f(x) = \log_2 x$.

a) $f(x) = -2^x + 2$

b) $f(x) = \sqrt[3]{x} + 1$

c) $f(x) = |\log_2(x + 2)|$



Domain: _____

'Zero': _____

The sign of function: _____

Domain: _____

'Zero': _____

The sign of function: _____

Domain: _____

'Zero': _____

The sign of function: _____

Exercise 3. (Apply)

Solve the inequality.

Use the signs of the functions for solving the inequality (the previous exercise).

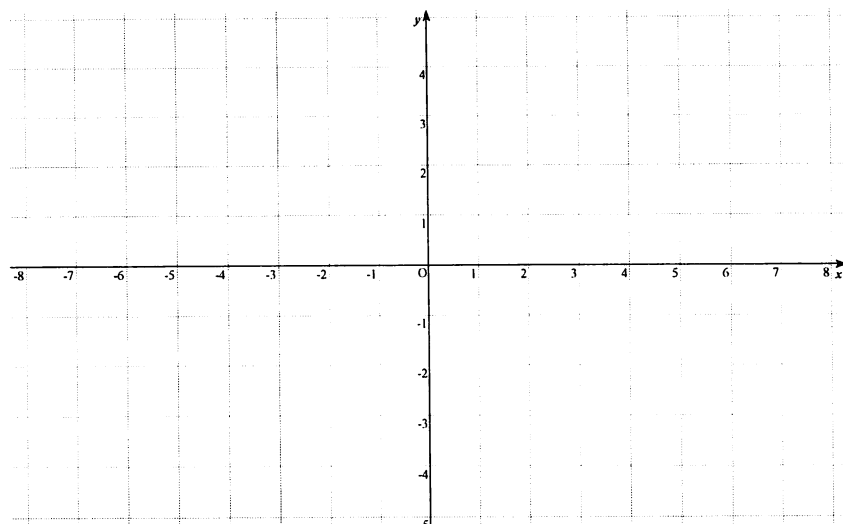
$$|\log_2(x + 2)|(-2^x + 2)(\sqrt[3]{x} + 1) > 0$$

The domain of expression in the inequality : _____

x			
$ \log(x + 2) $			
$-2^x + 2$			
$\sqrt[3]{x} + 1$			
$ \log_2(x + 2) (-2^x + 2)(\sqrt[3]{x} + 1)$			

The solution interval of the inequality : _____

Draw the graph of the function $f(x) = |\log_2(x+2)|(-2^x + 2)(\sqrt[3]{x} + 1)$ by using GRAPES. Check the solution of the inequality examining the sign of this function.



Exercise 4. (Review)

By using the graph of the function $f(x) = \sqrt[3]{x^3 + 3x^2}$, exam the properties of the function f :

- a) the domain: _____
- b) the function is equals zero for : _____
- c) the sign of the function:
 - a. $f(x) > 0$ for _____
 - b. $f(x) < 0$ for _____
- d)
 - a. the function is decreasing in _____
 - b. the function is increasing in _____
- e) The function has _____ relative extremums
(the number of)
- f) the relative extremums are: _____

- g) The function is (circle the correct answer)
 - a. 'pair'
 - b. 'add'
 - c. neither

Explain!

Exercise 6. (Apply)

Find the approximate values of the dimensions (the base radius and the height) of the cone with the largest (maximal) volume for the given “slant height”, $s=2\text{cm}$.

$$V(r)=\underline{\hspace{2cm}}$$

For $r \approx \underline{\hspace{2cm}}$ the cone, with the given “slant height”, $s=2\text{cm}$, has the largest (maximal) volume $V_{\max} \approx \underline{\hspace{2cm}}$

Exercise 7.

Find the approximate values of the dimensions of the cylinder with the smallest surface area for the given volume, $V=2 \text{ m}^3$.

$$A(r)=\underline{\hspace{2cm}}$$

For $r \approx \underline{\hspace{2cm}}$ and $H \approx \underline{\hspace{2cm}}$ the cylinder, with the given volume, $V=2 \text{ m}^3$, has the smallest surface area $A_{\min} \approx \underline{\hspace{2cm}}$.

Exercise 8. (Homework)

The cylinder with the largest area of the curved side surface (lateral area) is contained within the cone of height, $h=2$, and base radius, $r=5$. Find the approximate values of the dimensions of the cylinder.