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THE THINKING LEVELS OF FUNCTION BASED ON VAN HIELES' THEORY

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1. The purpose and the method of study

The purpose of this study is to describe the development of children's thinking about function. By this study, we can develop the process of learning for children which they can do mathematical activity on subject matters of function. For this purpose, we need the method of description. In this study, we use Thinking Level Theory by P.M van Hiele and Dina van Hiele. Thier theory descibe levels of language used by children in learning and thinking in geometry. If we know a level of some child, we will know his way of thinking. So we can develop the process of learning.

In this note, we describe the levels of function in Japanese student. The levels were found from the investigation by the way of van Hieles' theory.

2. Subject matter of function in Japanese curriculum

Some part of thinking development depend on development of knowledge. Development of knowledge about function depend on curriculum. So we need know curriculum in Japan. Following are examples, subject matter of function in Japanese curriculum.

In elementary school; Some real problems which solved by functional thinking and idea of proportion, writing graph. Formula of velocity, formula of area and etc. Formula is described with some words. In Japan, proportion and inverse proportion are thaught in 11 year old children, e.g. ...in direct proportion to each other.

In lower secondary school; Function is thaught from lower secondary sc... Variable and set, ...as a function of ..., graph of function are thaught. Function of ratio, linear function, $y=ax2$ are abstracted from real world.

In upper secondary school; Function is thaught with formula, e.g. $y=ax2+bx+c$, exponential function, logarithmic function, torigonometric function. Calculus is thaught.

3. Investigation in secondary school students

We investigated secondary school students' thinking in soloving problems, and analyzed their language from interviews. One of problems is follows.

" We had dropped the matter in the air. We measured the falling time from dropped and distance from dropped point to the matter. Following is the table of the measurment.

time (s)	0	1	2	3	4	5
distanse (m)	0	5	20	45	80	125

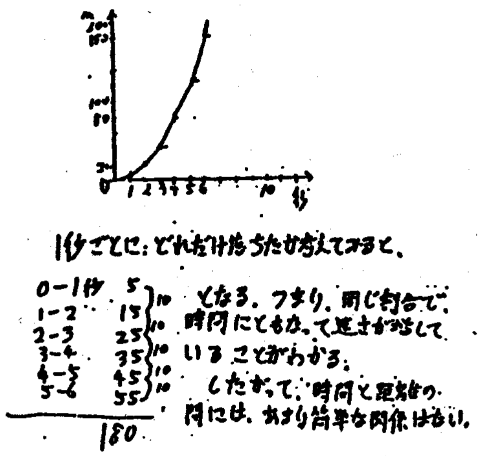
How do the matter fall? Describe the process of falling."

From analyzing students' language, we distinguished following three types of description.

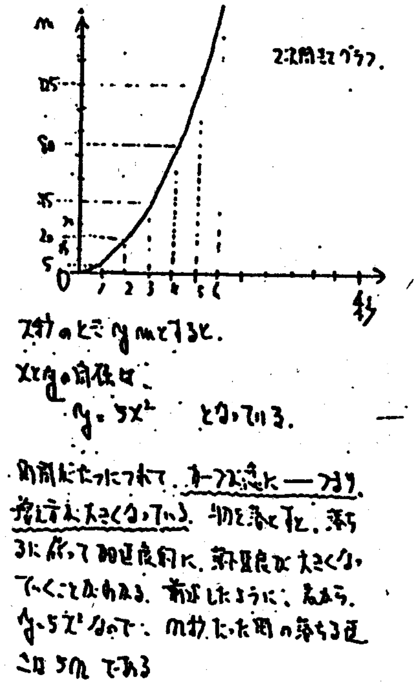
Type 1: In this type, students describe the number relations of table, e.g. distanse or speed increases 5, 15, 25, 35, 45. Some student discover formula, and draw graph. But they don't analyze the formula or graph.
Type 2; In this type, students analayze the formula y=5x2 and the graph, e.g. describing the increases from the graph.
Type 3; In this type, students analayze formulas and graphs by calculus.

Following are examples of three types.

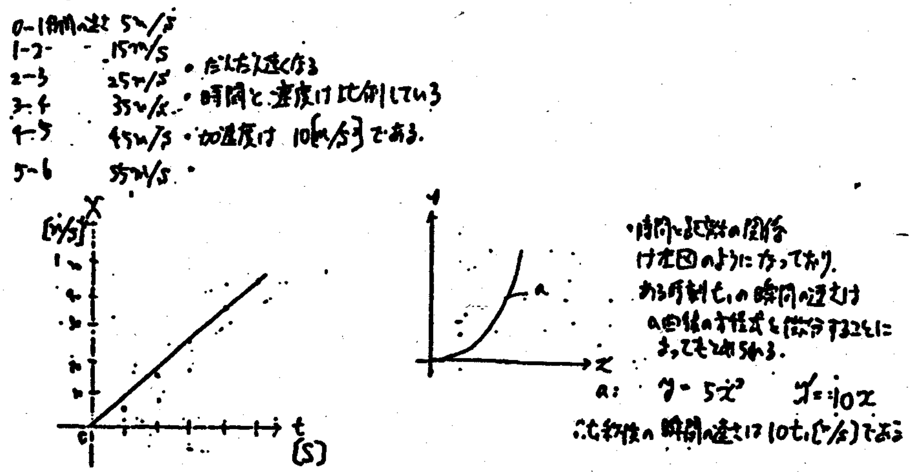
Type 1



Type 2



Type 3



4. Thinking Levels of Function

From the investigation of children's thinking, the research of the mathematical history and the analyzing of the curriculum, we distinguished the levels of function. The levels describe the development of thinking from the real world to caluculus. The levels are following.
Level 0; Concrete relations of a real situation, which are organaized by familiar language, are the subject matter. The familiar language isn't mathematical, but in many cases children use numerical language to explain some relations. For example, children often think that a heavy object falls faster than a light object, but they find it isn't truth by numerical experiment.
Level 1; The numerical relations which have been the organaizing devices at

the level 0 become the subject matter as the number relations on tables. The tables are organaized by means of the characters of number relations, e.g. proportion and inverse proportion. Children draw a graph only when they are given proper real situations. The main language consists of number, its relations and its calculations. For example, after a experiment of the falling object, children draw some tables and discover some characters of relations from them. Some children draw a graph in this context.

Level 2; The characters of number relations which have been the organaizing devices at the level 1 become the subject matter. The characters are organaized as the features of function by means of algebraic expression and graph. The main language is algebra, but the language contains geometry and analytic geometry. For example, when children explore some functions with algebraic expression they draw some tables and graphs. Then, they find the features of functions and explain them by algebra and geometry. When children explore the falling object, they use the knowledges of functions and describe the process of falling with some features of functions.

Level 3; The features of functions with an algebraic expression and a graph which have been the organaizing devices at the Level 2 become the subject matter. The features of functions are organaized by means of calculus. So the main language is calculus. Children explain the features of one function by the features of another functions such as the derived function. For example, the process of falling which was described by function is studied by defferential equations.

5. Implications for learning process

In this note, we describe the development of children's thinking about function by the levels. We can develop the effective learning process for children, only when we know the children's thinking, children's mathematical activity, and their development. H.Freudenthal said that the means of organaization of the lower level become a subject matter on the higher level. This is one of features of thinking levels and mathematical activity. In the levels of function, we can recognize this feature. For learning process of function, the levels of function implicate some aspects of children's thinking, children's activity, and their development. For example, when we recognaize children's level, we know the subject matter that they can learn, and that they need for development to the next level. If we don't know children's level, we teach them what they don't undrstand.

One of effective implication is that the levels indicate contexts of learning. Children can learn on the context which he undrestand. Children of Level 1 have the context that they think about numerical tables on real situations and they find some characters of number relations about them. So they can learn particular relational concepts such as proportion in this context. Children of Level 2 have the context that they study function. They can think about features of function with algebraic expressions, tables and graphs. For the development from Level 1 to Level 2, they have to change contexts of thinking. This change is the study of particular relational concepts on tables, algebraic expressions, and graphs. This is to give the children the new situations for thinking. Children of Level 3 have the contest that they study function by another functions such as derived functions. Studying method is calculus. The contexts change from Level 2 to Level 3 is the comparative

study of features on functions, e.g. comparing the graph of the function with tangential line of it.

Some aspect of thinking context depend on language of thinking. So the development of language imply the chainges of context. How do the language of thinking about function teach? We have to study this problem.

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