April 2009

Primary Education Improvement Project

Module 4: Numeracy

Maths for life

Provincial Workshop 4

- Currents Trends in Mathematics Education
- Creating Process Standards for mathematics Education
- The use of Blackboard in a mathematics lesson
- Alternative Planning strategy to support Learner-Centered-Instructions
- Teaching Mathematics through Problem Solving
- Introduction to geometry
- Professional Development of teaching mathematics through “lesson Study”.
- Poster Session

Ministry of Education
Port Vila, Vanuatu

Ministère de l’Education
Port Vila, Vanuatu

Compiled by Ian Kennedy, Primary Mathematics Tutor, VITE

Abbreviations

JICA – Japan International Cooperation Agency
JOCV – Japan Oversees Cooperation Volunteers
LCI – Leaner Centered Instruction
MOE – Ministry of Education
NUE – Naruto University of Education
PEO – Provincial Education Officer
VITE – Vanuatu Institute of Teacher Education
ZCA – Zone curriculum Advisors
VANTAM - Vanuatu Teachers' Association for Mathematics
PRIDE - Pacific Regional Initiative for the Delivery of Basic Education

CONTENTS

Acknowledgement

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10. My family and friends for moral support,

And to God, who made all things possible!

Introduction

The term numeracy came into existence in 1959 by a committee on education in the United Kingdom which said that 'numeracy' should 'represent the other side of the coin, “literacy”.(Crowther Report). Just as the early definitions of literacy have gradually progressed from ‘reading and writing’, numeracy now is more than ‘numbers and measurements’.

More recently in the eighties, the British Cockroft Committee developed a definition of numeracy. It stated that a numerate person should understand some of the ways mathematics can be used for communication, and this required the possession of two attributes:

1. being 'at-ease' with all those aspects of mathematics that enable a person to cope with the practical demands of everyday life and
2. the ability to understand information presented in mathematical terms.

As such, children in Vanuatu need to understand numeracy and the concepts of maths and how they can apply them to everyday life.

Because numeracy has to do with numbers and maths, what does it mean for our children?

Booker et al (1997) states that numeracy is:

> concerned with using, communicating and making sense of mathematics in a range of everyday applications; the ability to explore, hypothesize and reason logically and to use a variety of methods to solve problems.

To make it simple numeracy means children need to be able to use mathematics concepts in ways that benefit them in everyday life.

But how can we help children learn mathematics? Few people believe that there is an exact recipe for good mathematics teaching, yet most agree that the most vital ingredient is the teacher. An effective teacher must understand the mathematics to be learned, be sensitive to children’s needs and the ways they learn, and be familiar with a variety of strategies and methods of teaching.

The purpose of this module is to help you develop some prerequisites for being a good primary school mathematics teacher.

Being a good mathematics teacher is the result of experience, innovations, modifications flexibilities and acceptance of changes and challenges in mathematics education which are happening in the world today.

We often look for answers to our problems in teaching beyond the horizon but we never realize that as teachers we can collaborate together in each of our school to share and challenge our own problems. “Lesson study” is the main focus of this module where training is provided for teachers to tackle their own problems as collaborative and cooperative teams.

Unit 1
PEIP School Reports

Purpose
To inform project participants and staff of the activities, achievements, & issues that have occurred in PEIP participating schools, and create opportunities for dialogue and discourse on the impact of LCI, Assessment and Literacy on teaching and learning.

Objectives

- To strengthen the PEIP community through awareness, discussion, and reflection on LCI and assessment activities in schools.
- To provide opportunity for participants to share their experiences with their colleagues and provide suggestions to PEIP planners and staff.
- To create a record of PEIP activities for project managers.

Outcomes

- Short presentations from each participating school.
- List of activities, achievements, & issues resulting from work covered in the last two modules.
- List of strategies for overcoming issues & barriers.
- Completed School Report Forms for each school.
- Full group discussion on PEIP to date.
- Process Journal reflection on LCI, Assessment and Literacy.

Rationale
Workshops provide an opportunity for practitioners to share ideas, experiences, and challenges. With PEIP, each new Provincial or Local Workshop brings together the same group of teachers, and possibly new participants, to continue professional development as well as continue building collegial relationships, essential for establishing, building, and sustaining a community of practice.

Introducing new methodologies for teaching and learning brings new challenges to PEIP participants' professional practice. Activity 1 provides the opportunity for participants to share their experiences, discuss issues and pose strategies for maximizing the impact of the PEIP. Activity 1 also provides an opportunity for project managers to gain insight into the project's progress and begin discussion on strategies of improvement if needed.

Activity
Prior to Workshop 4, participating schools will be sent the PEIP school Report Form to guide participants reporting of the second phase of their project related work. Each school will complete the form to be handed in to PEIP staff, and prepare a 5 minute presentation of their school's PEIP activities. Participants should use the school report form to guide their presentation preparation. After the presentation, the facilitators should lead a full group discussion on presentation findings.

Notes to Facilitators
In order for participants to have adequate time to prepare their reports, the report forms should be sent to all schools prior to the workshop. If schools are unable to complete the report by workshop time, ask these schools to complete the form by the end of the workshop. Keep track of these using a checklist.
Unit one is the first unit of the workshop. Prior to the beginning, facilitators should review the workshop aims, and outcomes with the full group. School reports should start

immediately following the review. Each school can hand in their report forms as they present, or can be collected after if the forms are not complete.

As schools report their PEIP activities, the facilitators should keep a generalized running record and poster paper activities, issues, barriers, strategies, etc…This record will help to prompt the full group discussion.

Facilitators should regularly collect the participant's process journals to review their entries to gain insight into the progress of the project. Providing a suggestion box during the workshop can also be helpful as a learning tool and for feedback.

Unit 2
Trends in primary school mathematics

Purpose

Aware of the current trends in mathematics education in the world today.

Objectives

1. Describe several procedures and interactive styles of effective mathematics teachers
2. List several current trends in primary school mathematics.
3. Compare and contrast several trends in primary school mathematics from a personal perspective.
4. Update mathematics teaching to go along with global changes in mathematics education.

Learning Outcomes

1. Aware and participate in global changes in mathematics
2. Create an image of a good mathematics teacher

Rationale

As in-service teachers it is exciting time for you to be involved in mathematics education. With the invasion of technology and the expansion of a global economy, new and innovative ways of mathematical teaching are emerging. While we are still using isolated calculation and rote memorization of facts from a textbook, it is time to minimize such mode of teaching. The mathematical challenges of today extend beyond the "basics" and into the realm of complex, and contextual problem solving.
More than ever before, mathematics holds the key to success and productivity in our information society. This sends a very clear message to the educational community, and to teachers like you that in order to gain access to the opportunities of the future, students must have a sound understanding of mathematics. Those who do not meet this challenge will fall drastically behind.
This activity is designed to give you, the primary educator, a glimpse into the changing Landscape of mathematics and to offer effective instructional methods with which to manage change in your primary classroom.

Professor Ogura lectured on Trends of educational Improvement in the world at Tsukuba University, Japan, 2008

Activity 1 Question for Discussion (Participants) 30 minutes

Consider the following question for discussion for oral discussion.

1. Think back to your primary school years. How was mathematics taught? Outline a typical lesson. What was the role of the teacher? Of the child?
2. How do current primary school mathematics text books compare with those a few years ago?
3. What will be our strength and weaknesses in adopting new ideas in mathematics?
4. Are our current methods of teaching mathematics effective in some ways? Which method can be kept? Which one is to be abandoned? Why?
5. Share your experiences with the whole class.

Activity 2 (Participants) 20 minutes

Step 1: Form a group.
Step 2: Brainstorm on current trends of mathematics
Step 3: List down the characteristics that are needed to be a good mathematics teacher.

Activity 3 (Participants) Group Presentation 30 minutes

Step 1: Choose a presenter to present your list after completion.

Activity 4 (Participants) Creating an image of a good mathematics teacher. 90 minutes

Step 1: From the refined list provided by the Facilitator, recall five experiences you had in your mathematics class that helped you learnt mathematics.
Also think of five influences that negatively influence your mathematics learning.
Step 2: Use your list to develop an image of the kind of primary school mathematics teacher that you would like to be.

Activity 5 (Facilitator) Summary

Wrap up: The facilitator will summarize the main points learnt in this activity.

Note to facilitator:

Recalling your own mathematics instruction experiences are important because it can yield a general guideline for change. If your history of mathematics instruction has been favorable, then you can build new skills and content to enrich the solid base. However, even if you had negative experiences, you can resolve to avoid these erroneous styles of teaching and interacting. That alone can be corrective, but if in addition, new learning of mathematics can be undertaken, the result will be more positive.

Unit 3
Creating process standards for mathematics education in Vanuatu

Purpose

In order to participate fully as a citizen and a worker in our contemporary world, a person should be mathematically powerful. Mathematical power is the ability to explore, to conjecture, to reason logically, and to apply a wide repertoire of methods to solve problems. Because no one lives and works in isolation, it is also important to have the ability to communicate mathematical ideas clearly and effectively.

As such mathematics teachers should enable the students to develop and apply the created mathematical processes continuously through their development of content knowledge.

Objectives

1. To create process standards for mathematics education
2. Adopt or incorporate with NCTM standards and other standards
3. Being able to design mathematics tasks which align these process standards

Learning Outcomes

1. Aware of the importance of process standards in mathematics
2. Teach according to process standards

Rational

The four National Council of Teachers of Mathematics (NCTM) standards of problem solving, reasoning, communication and connections are goals interwoven throughout the mathematics standards. These goals are the reason why people study and use mathematics, and they should seep into everything we do in and outside the classroom. Whenever possible, mathematical learning should be placed in a broader, problem solving context and evaluated through performance assessments. In this setting, students discover questions involving numbers or equations from a real-world context which lead to answers that have meaning. Ultimately, all problems should be application problems; more ideally, problems should be presented in the broader context of an investigation or project. This way the students use problem solving, reasoning, communication and connections in every mathematical activity.

In Vanuatu we hardly take into consideration as to what process standards should be adopted or created for mathematics education. It is of great importance to think of creating process standards and takes into considerations the types of traditional or cultural mathematics already existed in Vanuatu and includes them.

If we are aware of the set process standards, the topics we teach in mathematics should be aligned with these standards. In that way we know that we are teaching ni Vans who will be able to solve any problems related to their daily living and in whatever circumstances

Activity 1 (Facilitator) 30 minutes
The facilitator will present a PPT about NCTM process standards in mathematics. An electronic copy of this presentation can be distributed to those who are computer literate or can use PPT presentation. Any other appropriate alternatives will be acceptable if electricity is not available.

Write your notes here

Slide 1

Process Standards for School Mathematics

NCTM

National Council of Teachers of Mathematics

Slide 2

PROBLEM SOLVING—The student will develop problem-solving abilities

- to build new mathematical knowledge;
- by analyzing problems that are routine and non-routine;
- by applying and adapting a variety of appropriate strategies (illustrating, guessing, simplifying, generalizing,etc);
- by monitoring and reflecting on the process of mathematical problem solving.

Slide 3

REASONING AND PROOF—The student will develop reasoning abilities

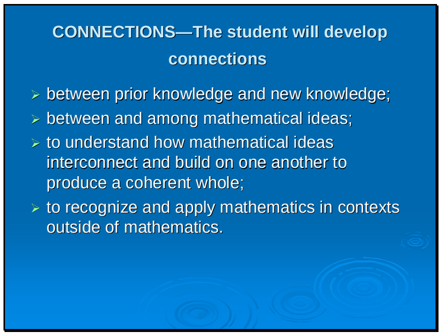
- to make and investigate mathematical conjectures and proofs;
- to evaluate information, mathematical arguments and proofs;
- to perceive patterns;
- to identify relationships;
- to formulate questions for further exploration;
- to evaluate strategies;
- to justify statements and defend work;
- to assess reasonableness of results.

Slide 4

COMMUNICATION—The student will develop written and oral communication skills

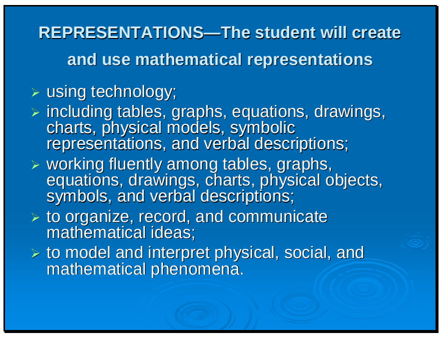
- to organize and consolidate mathematical thinking;
- to explain mathematical thinking coherently and clearly to peers, teachers, and others;
- to analyze and evaluate the mathematical thinking and strategies of others;
- to use the language and conventions of mathematical discourse (e.g., symbols, definitions, labeled drawings, and labeled answers) to express mathematical ideas precisely;
- to formulate logical arguments that clearly show why a result makes sense.

Slide 5



Write your notes here!

Slide 6



After the presentation, allow time for discussions, critiques and questions

Activity 2 (Participants)

Step 1: Get into your working group.
Step 2: Brainstorm on how to create your own process standards in mathematics
Step 3: As shown in the NCTM and Japanese process standards, design a similar one in your group.

Activity 3 (Participants)

Step 1: Choose a presenter to present your activity.
Step 2: Be prepared to answer questions and critiques from other groups. Be critical and creative in your responses.

Activity 4 (Participants)

Step 1: Design a lesson that will align some of these process standards.
Step 2: choose a member from your group to present the lesson in front of the whole group. Others will act as students.

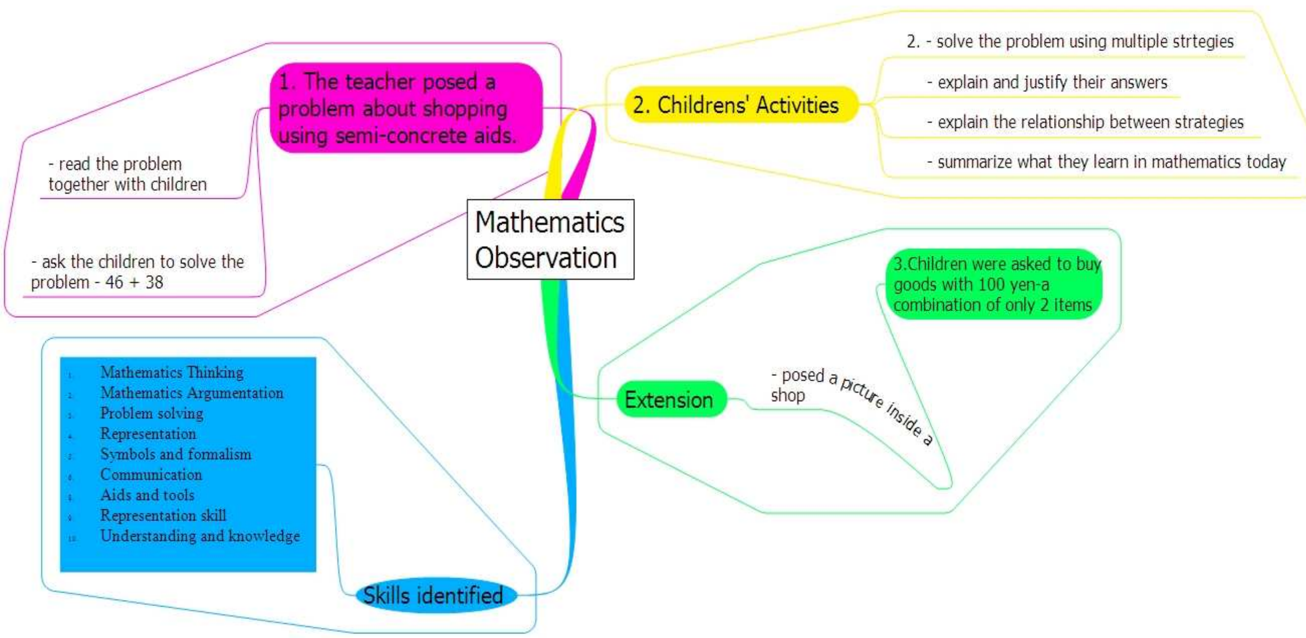
Note to facilitator: It is better to compromise and confirm provincial standards for mathematics. In this case, the facilitator with the participants can conclude on which process standards to adopt and target during mathematics lessons.

Activity 5 (Facilitator and participants)

Discuss with the participants how to plan lessons to address their process standards. Show another example of a lesson and align it with process standards created in activity 2

The diagram below shows a mind map (observation) of a class 3 math lesson in Tokyo during a lesson study in mathematics in 2008 by Ian Kennedy.

Note to facilitator:
Any other lesson previously observed or recalled by the facilitator can be used as an example instead of this one.



Activity 6 Facilitator and participants.
Show a video of a teacher teaching mathematics. Ask the participants to observe the lesson and list the process standards identified in the lesson.

Activity 7 (facilitator)
Wrap up: The facilitator will summarize the main points learnt in this activity.

Unit 4
Alternative planning strategy to support LCI

Purpose

To develop a concept map that supports a holistic approach to teaching and learning mathematics.

Objectives

- To draw a concept map for mathematics planning (mountain climbing method)
- To identify learning elements for a particular topic (strand) in mathematics and create a hierarchical order between each learning elements or sub-strands.
- To teach mathematics using the holistic approach.

Learning Outcomes

1. Arrange topics in hierarchical order
2. Creating a concept map of mathematics
3. Enable students to participate in planning their math activities

Rationale

Since the adoption of the Mathematics text books from Senegal, teachers find it hard to follow the suggested annual scheme in each of the text book especially the teachers' guide. The main reason is that the recommended scheme or planning grid dictates a fragmented method of teaching mathematics. While we may consider this as traditional or transmissive text books, switching to LCI does not support this idea. However to cater for a more logical planning, some teacher modifies the scheme to their own suits.
The use of concept maps as a teaching strategy was first developed by J. D. Novak of Cornell University in the early 1980's. It was derived from Ausubel's learning theory which places central emphasis on the influence of students' prior knowledge on subsequent meaningful learning.
According to Ausubel, "the most important single factor influencing learning is what the learner already knows. Thus meaningful learning results when a person deliberately and plainly ties new knowledge to appropriate concepts they already possess. Ausubel suggests that when meaningful learning occurs, it produces a series of changes within our whole cognitive structure, modifying existing concepts and forming new linkages between concepts. This is why the analysis of the textbook is of vital importance.

Activity 1 (Participants) Question for Discussion (10 minutes)

Consider the following question for oral discussion.

1. What are the constraints of the original panning in the teachers' guide?
2. What can be done to address these constraints?
3. What have you done personally to overcome these constraints?
4. Why do we need a concept map for mathematics planning?
5. Are there any other alternatives?

Activity 2 (Facilitator) Presentation 20 minutes

Facilitator to present a summary of concept map using the mountain climbing method. If electrical power is available, the facilitator can use the PowerPoint presentation already prepared or otherwise use appropriate alternatives such as using flips charts

Slide 1

Overview of this Presentation

Summary of the Mountain Climbing Method
- aims of the theory
- cognitive differences in students and the teacher structural systematic and logical arrangement
- the implication of this theory in other countries (Kenya, Africa)
- How to implement the theory
- Group Activity
- Group Presentation

Write your notes here!

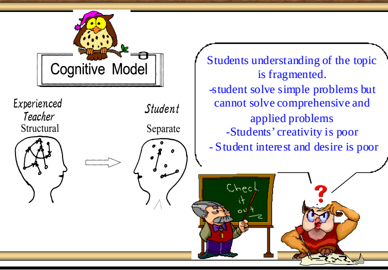
Slide 2

Aims of the Mountain Climbing method

1. to have students understand the contents of individual learning elements to a satisfactory degree,
2. to have them grasp and understand the relationship among the learning elements, and
3. to have them grasp and understand the relationship between a part and the entirety of the learning contents so that they may see the overall structure of those contents.

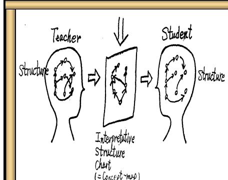
Slide 3

Cognitive differences between the teacher and the student



Slide 4

The cognitive implications of Mountain Climbing Method in Mathematics



- Establish link between elements of a topic to enhance a coherent understanding
- Solve applied and comprehensive problems
- Activate creativity
- Increase desire and interest

Write your notes here!

Slide 5

The implication of Mountain Climbing method in Kenya, Africa

Method:
1. Group A Traditional Method
2. Group B Mountain Climbing method

	Post achievement test	Pre achievement test	Difference
Group A	52.8	51.2	1.6
Group B	78.2	37.0	42.2

Slide 6

How to put this theory in practice

Choose a unit or topic from your text books

(Tr guide, pupils book, the syllabus and other relevant resources.) and select 10/20 or less learning items or elements

1. Connect the elements with each other as shown in fig 1
2. Reconsider and correct the structure if there are mistakes
3. Refine until you complete a concept map

Slide 7

The teaching procedures

At the beginning of the lesson hand out the;

- concept map,
- Reasoning arrow lines and
- Self diagnosis sheet

Slide 8

The teaching procedures con't

Students fill in the concept map and ;

- Explain the learning elements
- Fill in formulae and examples
- Self made problems and answers

Put all the learning elements in hierarchical order

Teacher: provides help to ensure that the students understand the meaning of each learning element and place them in a hierarchical order . (Fig 4) Link

Write your notes here!

Slide 9

Teaching procedures con't

- Students fill in the table of reason

Tr. Helps the students understand clearly the difference and interrelation of the learning elements (fig 5)

- Students may write what they feel strange or wish to study deeper in the self- diagnosis sheet (fig 6)
- Homework – enable students to discover new and valuable themes

Slide 10

The teaching procedures

At the completion of the last chapter;

- Reconsider the concept map
- Discuss the whole structure

The teacher aims to make students elaborate on learning elements and understand more about the structural relations among whole learning elements

- Enable the students to think structurally and creatively.*

Slide 11

The teaching procedures

Upon completion of the unit,

- collect the self diagnosis sheets,
- organise students in groups according to their responses and;
- Assign a research topic for them to cultivate the ability to solve problems by themselves

Upon completion, the students will give oral presentation in class and discuss their achievements/solutions of the assigned task.

Slide 12

Group Activity

- Sort out and arrange the learning elements in hierarchical order

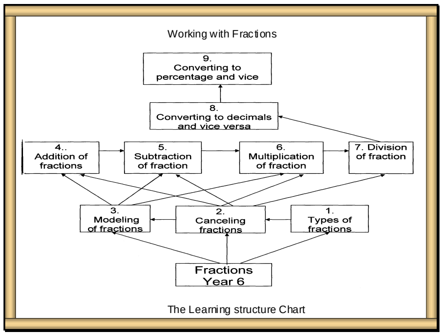
Follow the steps in slide no. 7

As students describe each learning element by inserting formulae or operational procedures in each learning element

Present your work and compare with other groups Link

Write your notes here!

Slide 13



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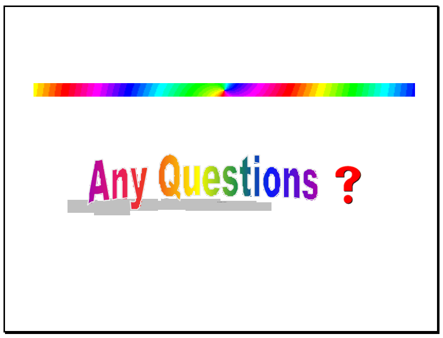


After the Facilitator's presentation, divide the participants into groups of teaching classes and each group will choose a math topic to work on.

Activity 3 (Participants) 120 minutes

Step 1: Form topic groups established by your facilitator or of your choices.
Step 2: From your suggested topic in mathematics, study your class' text books and other relevant sources.
Step 3: List all the sub-strands, or learning elements for your topic. For easy arrangement use manila clips
Step 4: Once you have studied your topics, create a concept map
Step 5: Spend some times reflecting on your map and make changes if needed.

Activity 4 Presentation 2 (Participants) 60 minutes

After the completion of the activity participants will choose a presenter from their groups to present their concept maps.
Questions, critiques and oral discussions can take place after each presentation.

Activity 5 (Facilitator) 10 mins
Summary
The facilitator will summarize today's activity and emphasize the main points.

Unit 5
Effective use of the blackboard in a math lesson

Purpose
This unit targets how teachers should use the blackboard effectively during their math lesson

Objectives
- Demonstrate the effective use of the blackboard during a math lesson
- Aware of the impact of good blackboard use in alignment with process standards
- Practice the effective use of the blackboard in a lesson practicum
- Aware of the impact that will enable the children to participate in oral presentations and discussions

Learning outcomes
- Use the blackboard effectively to promote learning

Rationale

In Vanuatu, teachers often overlooked at the importance of blackboard management during math lesson. Normally teachers just write or scribble any math topic anywhere on the board while children do the same. The writing can be rubbed off anytime they want during the lesson.
This unit deals mainly with effective use of the blackboard and inform us of the importance of keeping the board clean and writing our math lesson in a logical and sensible manner to enhance students learning and understanding of math concepts. Furthermore it will help the children to link their ideas with their colleagues and the teacher.
A Research by Makoto, a Japanese educator gives us an awareness of how we can use the blackboard effectively during our math lesson. It will be a guide to us teachers to maximize good and effective use of the board during our math lesson.

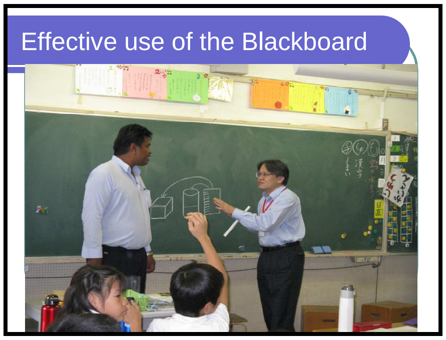
Activity 1 (Participants) 10 mins
Take some time to discuss among your group members how each of you manages his/her blackboard during any math lesson.

Activity 2 (Facilitator and participants)
After the participants' discussions on how they use the board, ask some of them to share their experiences. Some of these practices could be integrated in this unit.

Activity 3 (Facilitator)
Present a PowerPoint on effective use of the board. Use other appropriate means if electrical power is not available.

Write your notes here

Slide 1



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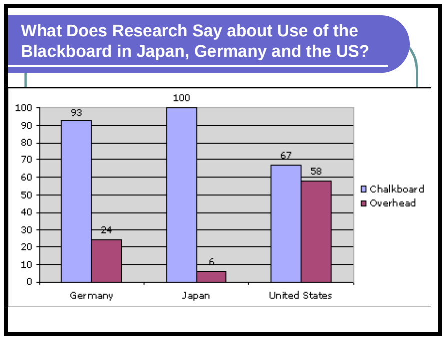

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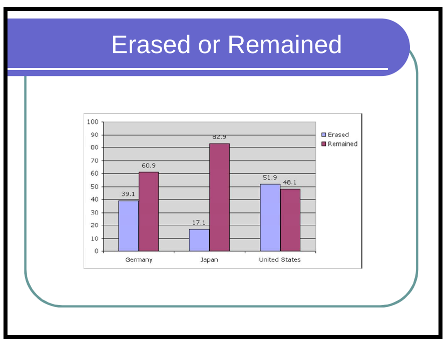


Slide 4

(1) Keeping a record of the lesson

- As TIMSS research indicated (Stigler, et al., 1999) the blackboard is used for keeping a record of the lesson.
- the actual story problem the class worked on during the lesson;
- the main questions that provoked student thinking; student voices, opinions, and things noticed; various solutions determined by the students; questions and decisions resulting from student discussions; and important mathematical ideas generated by discussions.
- useful when the teacher wants to refer to something that happened or was discussed earlier in the lesson.
- By looking at the blackboard during and after the lesson, students can gain a lot of information, which will go a long way to help them make sense of what they are learning.

Slide 5

(2) Helping students remember what they need to do and think about

- Keeping the story problem, directions, tasks, and questions on the blackboard
- provides a place for students to check what they are supposed to do.
- If students forget what to do or what to answer while they are engaged in the learning activity, they can simply look at the blackboard to obtain necessary information to get back on track.
- Students can also refer each other to what is on the blackboard to help each other.

Write your notes here!

Slide 6

(3) Helping students see the connection between different parts of the lesson and the progression of the lesson

- A well-organized blackboard documents coherence of the lesson.
- At the end of the lesson, it shows a coherent flow of the lesson, which in turn helps students to see the logical connections among all parts of the lesson.
- It also shows how the lesson progressed, how student ideas were incorporated into the lesson, and
- how the conclusion of the lesson was reached.

Slide 7

(4) Contrasting and discussing ideas students present

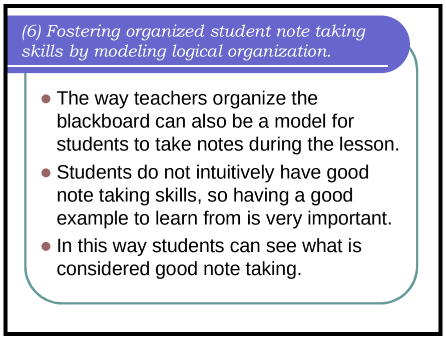
- a place for students to discuss the presented ideas.
- the similarities and differences in ideas are determined.
- In addition, the merits of using a certain method to solve the problems are also discussed.
- Through those discussions, the students might develop new ideas or questions they want to investigate. I call this type of blackboard use a “collective think-pad” because a whole class discussion is carried out based on the ideas presented on the blackboard. (Makoto, 2003)

Slide 8

(5) Helping to organize student thinking and discover new ideas.

- Used for manipulating presented materials to help organize student thinking and discover new ideas. For example, sorting, lining up, categorizing, and moving directions can be helpful for students to think about, discover, and discuss new ideas.
- Teachers can facilitate the discussion and help them think about important mathematical ideas.
- Combined with the discussion on the similarity and differences of the presented students’ ideas, is critical for teachers to skillfully carry out child-centered or discovery-oriented lessons.

Slide 9



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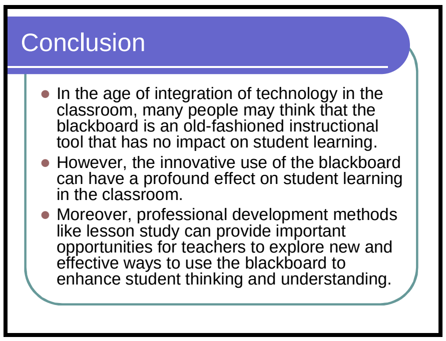

Activity 4 (Facilitator)

Teach a lesson in mathematics using the blackboard to demonstrate the effective use of it as. (Choose any topic at all) Use LCI methodology.

Ask the participants to participate as children and at the same time observe the lesson and comment for further improvement. Ask them to make some alignments with standard processes previously developed.

Unit 6
Teaching mathematics through Problem Solving

Purpose
The main purpose of the "teaching mathematics through problem solving" approach is to help students develop a deep understanding of mathematical concepts and methods by engaging them in trying to make sense of problematic tasks in which the mathematics to be learned is implanted.

Objectives:

- Differentiate between teaching problem solving and computational exercise.
- Identify characteristics of good problem solvers
- Apply a broad range of problem solving strategies to solve a variety of problems
- Create classroom environment that encourages children to effectively solve problems
- Create problems to supplement elementary school text book problem solving experiences
- Classification of mathematics problems

Learning Outcomes

1. Being able to write different types of problems in mathematics and categorize them
2. Provide resources and good learning environment for students to solve problems

Rationale

Problem solving is the cornerstone of mathematics instruction. Students must learn the skills of effective problem solving, which include the ability to:

- read and analyze a problem
- identify the significant elements of a problem
- select an appropriate strategy to solve a problem
- work alone or in groups
- verify and judge the reasonableness of an answer
- communicate solutions

Acquiring these skills can help students become reasoning individuals able to contribute to society.

As students move through the grades, the curriculum presents them with increasingly diverse and complex mathematical problems to solve. To encourage students' abilities to communicate, explore, create, adjust to changes, and actively acquire new knowledge throughout their lives, mathematical problem solving should evolve naturally out of their experiences and be an integral part of all mathematical activity. Effective problem solving consists of more than being able to solve many different types of problems. Students need to be able to solve mathematical problems that arise in any subject area and to draw upon skills developed in more than one area of mathematics. Becoming a mathematical problem solver requires a willingness to take risks and persevere when faced with problems that do not have an immediately apparent solution.

Activity 1 (Facilitator) 60 minutes

The facilitator will introduce problem solving by providing a brief introduction on the importance of teaching mathematics through problem solving, a major paradigm shift in mathematics education.

Then he will give a presentation that highlights the main components of problem solving. If electrical power is not available, use other appropriate means.

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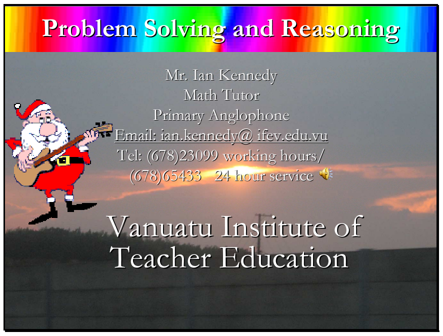

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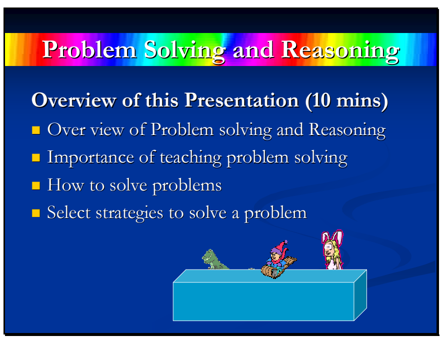

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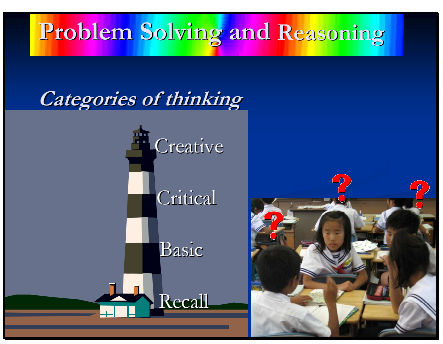

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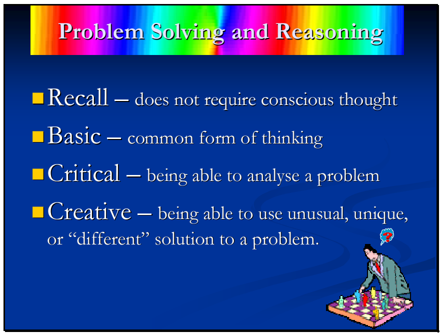

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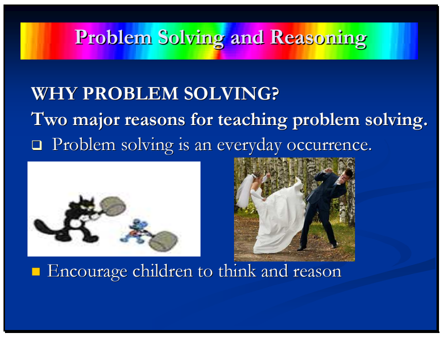

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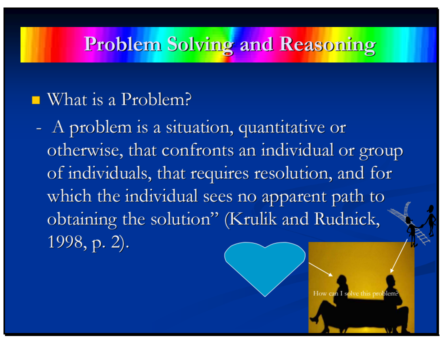

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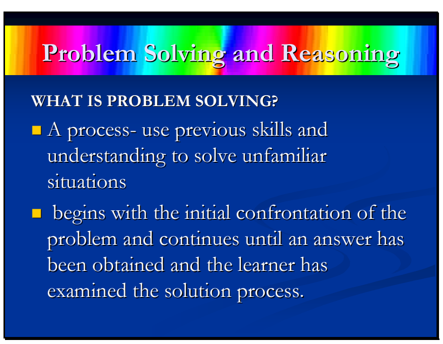

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Slide 8

Problem Solving and Reasoning

The goal of teaching mathematics has two parts:

1) help students learn facts, master skills, and obtain information;
2) help students acquire the ability to use these facts, skills, and information in solving problems and developing their reasoning skills.

The latter is most important: the skill of problem solving and reasoning.

Slide 9

Problem Solving and Reasoning

How To Solve a Problem

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back.

(George Polya, 1945 p 93)

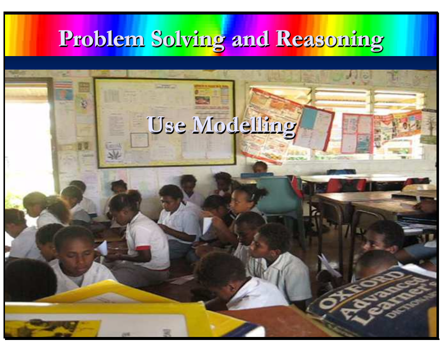
Slide 10

Problem Solving and Reasoning

Some Problem Solving Strategies

Look for a pattern	Draw a diagram
Guess and test	Use logical reasoning
Work backward	Write an equation
Reduce and expand	Solve a simpler problem
Act it out	Simulate or experiment
Make a table	Exhaustive listing

Slide 11



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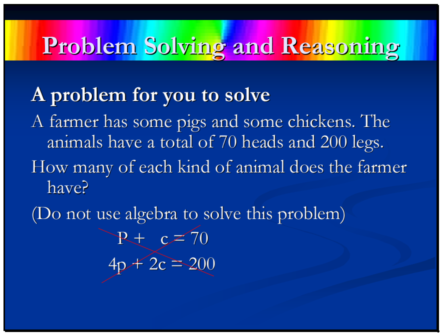

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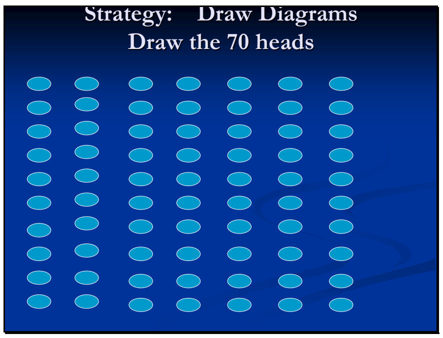

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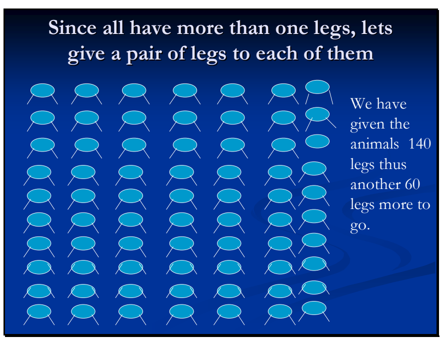

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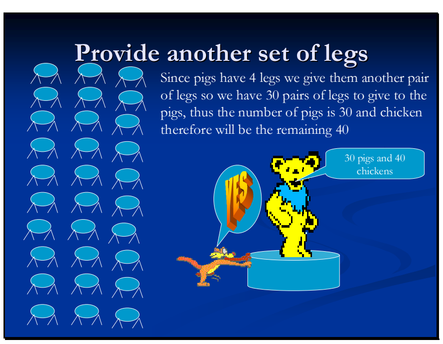

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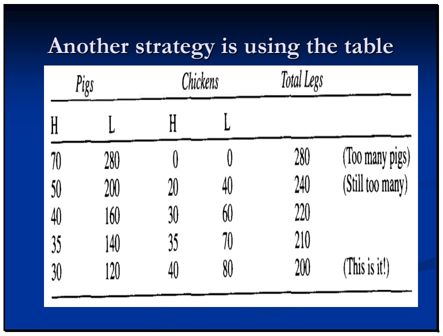
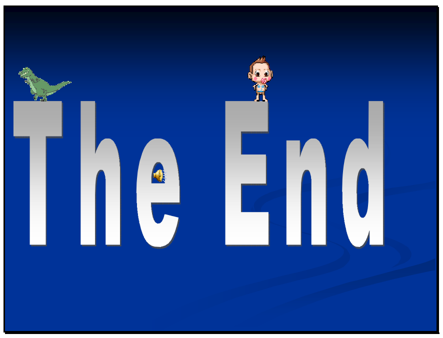
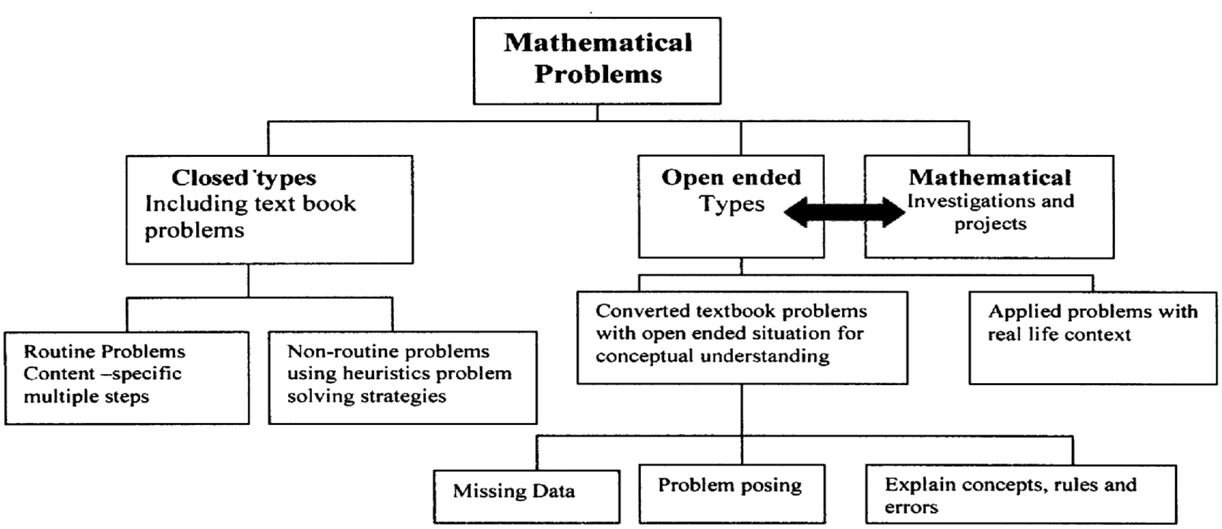


Pigs		Chickens		Total Legs	
H	L	H	L		
70	280	0	0	280	(Too many pigs)
50	200	20	40	240	(Still too many)
40	160	30	60	220	
35	140	35	70	210	
30	120	40	80	200	(This is it!)

Slide 17



Classification of mathematics problems



Adapted from Foong Pui Yee
National Institute of Education, Singapore

Activity 4.1 (Participants)   90 minutes

Step 1: Form groups established by your facilitator or of your choices.
Step 2: Together, study the diagram by Foong Pui Yee on characterization of problems. Read additional notes at the back of this manual to help you.
Step 3: Discuss and design 4 different types of problems;

I. routine problem
II. non routine problems
III. Open ended problem
IV. Investigations and projects

Step 4: Discuss their differences in your group before presenting it to the whole class.

Activity 4.2 (Participants) Group Presentation 45 mins

Step 1: Choose a presenter to present your problems. Discuss the differences between each of them.
Step 2: Other members of the group must be ready to answer any critiques or queries from other groups.

Activity 4.3 (facilitator) Summary 15 mins
The facilitator will summarize today's activity and emphasize problems that yield high order level thinking.

Unit 7
Professional development of teaching mathematics through "Lesson Study"

Purpose
Introduce effective mathematics teaching through Lesson study

Objectives
- To make explicit the key elements of lesson study
- To show groups working through challenges that arise in mathematics
- To support an unrelenting focus on student thinking and learning
- To create lesson study groups within each school.

Learning outcome

- Understand the main features of lesson study
- To create lesson study groups within each school
- Share ideas and skills with other teachers through lesson critiques

Rationale

Lesson study is a professional development process which Japanese teachers engage in to systematically examine their practice, with the goal of becoming more effective. This examination focuses on teachers working collaboratively on a small number of "study lessons". Working on these study lessons involves planning, teaching, observing, and critiquing the lessons. To provide focus and direction to this work, the teachers select an overarching goal and related research question that they want to explore. This research question then serves to guide their work on all the study lessons.

While working on a study lesson, teachers jointly draw up a detailed plan for the lesson, which one of the teachers uses to teach the lesson in a real classroom while other group members observe the lesson. The group then comes together to discuss their observations of the lesson. Often, the group revises the lesson, and another teacher implements it in a second classroom, while group members again look on. The group will come together again to discuss the observed instruction. Finally, the teachers produce a report of what their study lessons have taught them, particularly with respect to their research question.

The lesson-study process has an unrelenting focus on student learning. All efforts to improve lessons are evaluated with respect to clearly specified learning goals, and revisions are always justified with respect to student thinking and learning.

--Stigler, J.W., & Heibert, J., *The Teaching Gap*, p. 121

Activity 1

Question for Discussion (10 minutes)

Consider the following question for oral discussion.

1. What do we usually do when we find some topics in mathematics hard to teach?
2. How can we improve our mathematics lessons using our own teachers as expertise?
3. Which topic is difficult to teach? How do you address this difficulty?
4. After teaching a particular topic what are some observable behavior displayed by our children?

Activity 2   Presentation   20 minutes

To make explicit the key elements of lesson study the facilitator will present a summary of “Lesson study” using power point presentation. If electrical power is not available the use of kitchen paper or overhead projector are appropriate as well. Just list down the key elements and present it in the simplest form.

Write your notes here!

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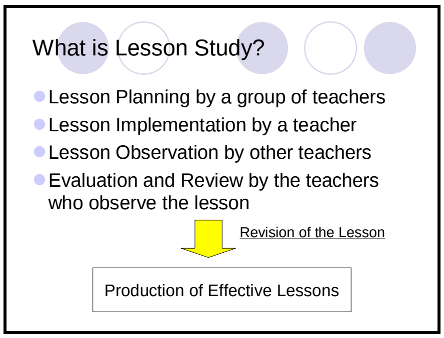

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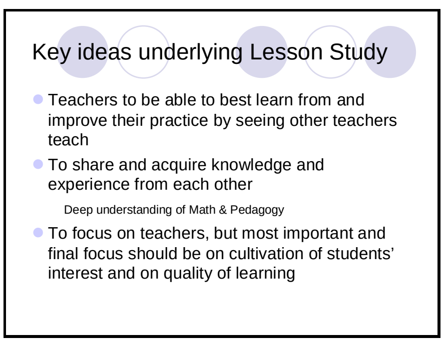

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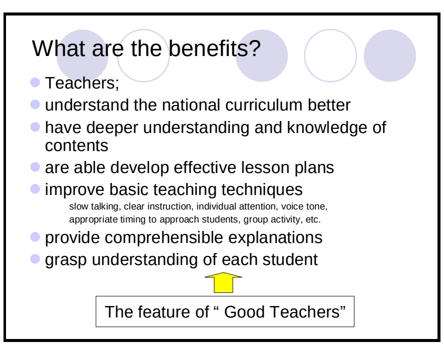

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Slide 4

Process of Lesson Study

Preparation (1) (2) (5)

Actual Lesson (3) (6)

Review (4) (7)(8)

(1) Problem identification
(2) Class planning
(3) Class implementation
(4) Class evaluation and review of results
(5) Reconsideration of class
(6) Implementation of class based on reconsiderations
(7) Evaluation and review
(8) Sharing of results

All of these processes are performed in collaboration with other teachers.

Slide 5

"Before It's Too Late"

James Stigler:
A researcher at UCLA
Videotape research studies of mathematics teaching

" The key to long-term improvement in teaching is to figure out how to generate, accumulate, and share professional knowledge."

Lesson Study has proved to be one such means.

Slide 6

Process of Lesson Study

1. Preparation: Planning

(1) To put the intention of the National Curriculum into a curriculum implemented in the classroom
(2) To select topics relevant to the purpose of the lesson
(3) To refining the lesson design based on the actual needs of students
(4) To prepare for teaching materials
(5) To tie all of the information together into a lesson

Collaboration with other teachers

Slide 7

Process of Lesson Study

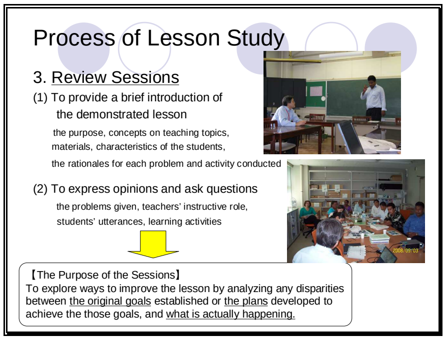
2. Demonstration

(1) To conduct a lesson based on the lesson plan devised

(2) The lesson to be observed by many teachers

Write your notes here!

Slide 8



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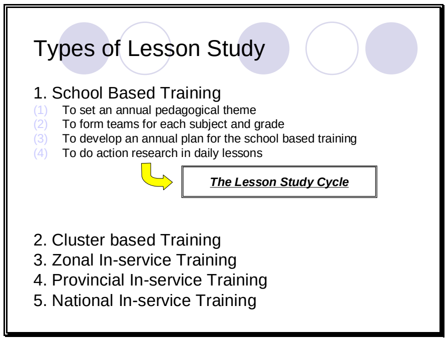

After the Facilitator's presentation, divide the participants into groups of teaching classes and each group will choose a topic to plan on.

Activity 5.1 6 hours

Step 1: Form groups to plan a lesson together.
Step 2: Identify a mathematical problem create a research question or problem from a realistic context. E.g. From one of your members' experience; choose a participant from a nearby school.
Step 3: Follow the same lesson plan template from the previous workshop. A copy is included at the back of this manual
Step 4: Choose a member of your group to present your lesson. If time allows you practice your lesson in front of your group before finalizing your lesson plan. Make necessary modifications and innovations as needed.

Activity 5.2 Teaching practicum

Step 1: Teach the lesson in a normal class
Step 2: Fill in the observation form as you observe.

After the presentation of the lesson, let the teacher talked about what he or she experienced before, during and after the lesson. Critiques should be carried out in a constructive manner after that.

Activity 5.3

Step 1: Refine the lesson plan and share with other members of the group.
Step 2: Write a report of your lesson.

Summary

The facilitator will summarize today's activity and emphasize the main points to be considered when carrying out lesson study

Unit 8
Introduction to geometry

Purpose

- The purpose of this activity is to provide information to participants about the different stages or hierarchical arrangement in geometry teaching.
- Implication of the van Hiele geometrical thought in this arrangement.

Objective

- Understand different levels of learning geometry according Van Hiele taxonomy
- Align with mathematics curriculum
- Design or create lessons according to the Van Hiele geometrical thought.

Learning outcomes

- Understand the arrangement of topics in geometry according to Van Hiele geometrical thought
- Teach according to Van Hiele geometrical thought
- Design extra geometrical activity according to Van Hiele

Rationale

Geometry is the study of shape and size. Geometry was probably first developed to measure the earth and its objects. The word "geometry" *geo* + *metry* means "**earth** + **to measure**."

Learning geometry typically begins at a very young age and develops into a formal investigation of the subject as the years pass.

Dina and Pierre van Hiele were Dutch teachers of mathematics. In the 1950s, they conducted research on how people develop geometry knowledge. The van Hieles found that geometric understanding is developed by progressing through 5 distinct levels. In order to pass from one level to the next level, students must master the knowledge at their current level of understanding.

The 5 levels of this development are visualization, analysis, informal deduction, formal deduction, and rigor. This workshop will show a progression of appropriate skills and activities necessary for children to learn geometry. This is to ensure that children study geometry according to their levels.

Children initially should be exposed to geometric concepts through the use of manipulatives or concrete activities. Students should then move into using both manipulatives and pictorial representations, eventually moving to the abstraction of numerical form. Some students will move away from manipulatives quickly while others will need more time to grasp the concepts. Only after students have mastered the concepts through the previous forms of representation, will they be ready to develop and understand formulas.

At the conclusion of the workshop, we will revisit the Van Hiele levels and discuss activities appropriate to each of the levels.

Activity 1 (Facilitator)

Present a PPT on the Van Hiele geometrical thought. If electrical power is not available, use kitchen paper and copy the van Hiele level and present it to the participants.

Write your notes here!

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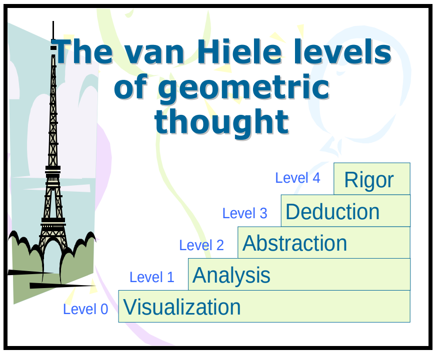

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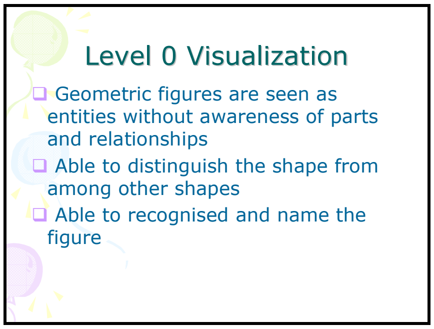

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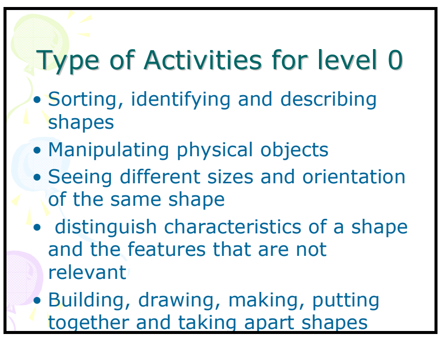

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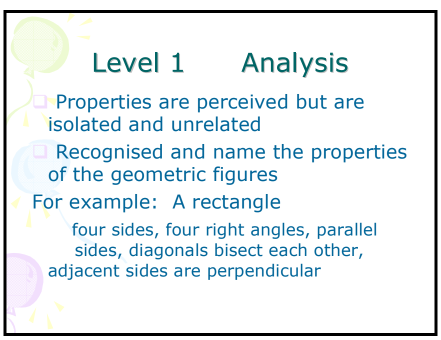

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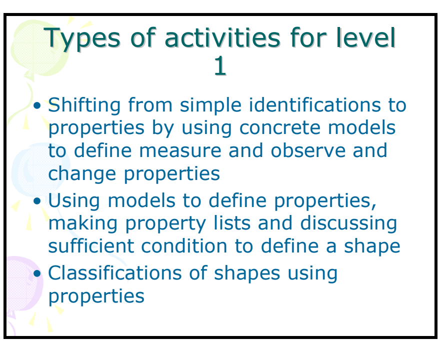


Write your notes here!

Slide 6

Level 2 Abstraction

- Definition are meaningful
- Relationships are perceived between properties and between figures
- logical implications and class inclusions are understood
- role and significance of deduction is not understood

Eg. I know it's a rectangle because it is a parallelogram with four right angles

Slide 7

Types of activities for level 2

- Using models and property lists, and discussing which group of properties constitute a necessary and sufficient condition for a specific shape
- Using informal, deductive language ("all," "some," "none," "if-then," "what if," etc.)
- Investigating certain relationships among polygons to establish if the converse is also valid (e.g., "If a quadrilateral is a rectangle, it must have four right angles; if a quadrilateral has four right angles, must it also be a rectangle?")

Slide 8

Level 3 Deduction

- can construct proof
- understand the role of axiom and definitions
- know the meaning of necessary and sufficient conditions
- able to supply reasons for steps in a proof

Slide 9

Level 4 Rigor

Very abstract geometry-The majority of high school geometry courses is taught at Level 5.

Slide 10

Types of activities for level 4 and 5

- Very abstract geometry-The majority of upper secondary school geometry courses is taught at Level 4 and 5. almost taught without models or manipulatives

Activity 2 (Participants) 60 mins

Step 1: Get into your group
Step 2: Design geometric activities suitable for each level.
Relate these levels to the activities found in the text books.
Step 3: Present your work to the whole group.

Activity 3. (Facilitator) 60 mins

The facilitator will share with you different levels of geometrical thought using a variety of teaching resources.

Unit 9
Group Poster Session

Purpose
To develop a poster presentation of activities covered throughout this week for peer review, critique, and discussion

Objectives
- To organize and prepare a poster presentation of all activities covered
- To provide constructive feedback on other groups' activities

Outcomes
- Presentation of all the activities
- Group assessment critiques

Rationale
What has been covered this week is so much in terms of teaching methodologies to support LCI. Participants have been working hard to produce very good presentations and have come up with collective ideas to integrate with exiting ones, those proposed by this module. Participants have not worked in isolation, but through on-going collaboration among group members. Every aspect within each unit has been discussed, reviewed, and revised through dialog and discourse among colleagues. The workshop is a model for project-based learning and illustrates how the social aspects of learning can greatly enhance the quality of the outcomes. A new concept in professional and community development is called a Community of Practice. A community of practice is any group of people who agree to work together to improve some aspect of their lives. Communities of practice learn from doing. Dialog and discourse among community members is essential for improvement and growth. Communities of practice have over-arching values of respect, tolerance, constructive criticism, and sharing the labours when tasks need to be done. This is an emphasize of lesson study; improving mathematics education through collaborative work.

Questions for Discussion
1. What have you learned from the peer assessment process used in this workshop?
2. What is the value of collaboration with colleagues for professional development?
3. What are the challenges of collaboration?
4. How does a poster session help to improve teaching practice?
5. What changes have you seen in your professional perspective since the beginning of the workshop?
6. How would you use a poster session in your teaching?

Activity 1 (Participants)
Develop a Poster Presentation for “ maths for life”
Try and link all the activities together to create a wholesome idea of all the activities covered this week.
1. Maximum of four (4) sheets of poster paper.
2. Use either English, French, or Bislama
3. Writing should be large enough to read standing a few meters away.
4. The names of the group members and their schools should be included.

Activity 2 (Participants)
Fill in the critique form while observing other groups' posters. The facilitator will distribute these forms before observation

Appendices

Appendix 1 PEIP SCHOOL REPORT

PEIP School Report Form

Please complete the following form to be submitted to PEIP staff at the upcoming Provincial Workshop. Use the back of this paper if you need more room. Please work together with your PEIP partner to complete it.

Name of Participant: ________________________

Island:	**School:**	**Date:**

Name of **PEIP** Activities and Achievements

Please provide information on PEIP related activities and achievements at your school by using the list below: Please include numbers of multiple activities.

Government/trained teachers observation : ____

____

____

____

Untrained teachers Observation: ____

____

____

6. Community awareness: ____

____

____

____

____

7. Teaching LCI lessons: ____

____

____

____

____

8. Implementing Module 2 [Assessment]: ______________________________________

____

____

9. Implementing Module 3 [Literacy]:

10. Training other teachers: ____

____

11. Nearby school workshops: ____

____

____

12. Developing lesson plans for Literacy: ______________________________
____

____
____
____

Informal discussions with students, teachers, parents, community members: ______________
____
____
____
____

PEIP- related Issues, Barriers, and Challenges

Please list and explain any issues, barriers, or challenges that have occurred as a result of PEIP, and any strategies that you have used to overcome the problems. Areas that might be covered are: conflict with other educators, students, parents, or community; lack of supplies or resources; transportation & communication; curriculum conflicts, gender issues, equity issues, etc. Be sure to include your strategies of approach!

Issues	Barriers	Challenge

Suggestions for PEIP

Please provide PEIP staff with any suggestions you have for the program

Appendix 2
A complete list of characteristics which make a good mathematics teacher

1. Willingness to be flexible, to be direct or indirect as the students demand.
2. Ability to perceive the world from the students' point of view.
3. Ability to personalize their teaching.
4. Willingness to experiment.
5. Skill in asking questions.
6. Knowledge of subject matter and related areas.
7. Provision of well-established examination procedures.
8. Provision of definite study help.
9. Reflection of an appreciative attitude (using nods, smiles, comments).
10. Use of conversational manner in teaching—an informal, easy style.

Appendix 3
The Theory of the Mountain-Climbing Learning Method

1. The significance and purpose of the Mountain-climbing learning method

We will make clear what the aims of the Mountain-climbing learning method are and what skills and abilities in students we aim to enhance.

— We believe that by expressing our purpose clearly, the path of teaching will become visible of its own accord. If we leave our purpose vague, we will only confuse our students' thinking and make it difficult for them to understand learning materials. —

1.3 The significance and purpose of the Mountain-climbing learning method

(1) The cognitive structure of veteran teachers and students - The importance of structuring

Let us first examine the cognitive structure of veteran teachers and students. Figure 2 shows a model of the structure.

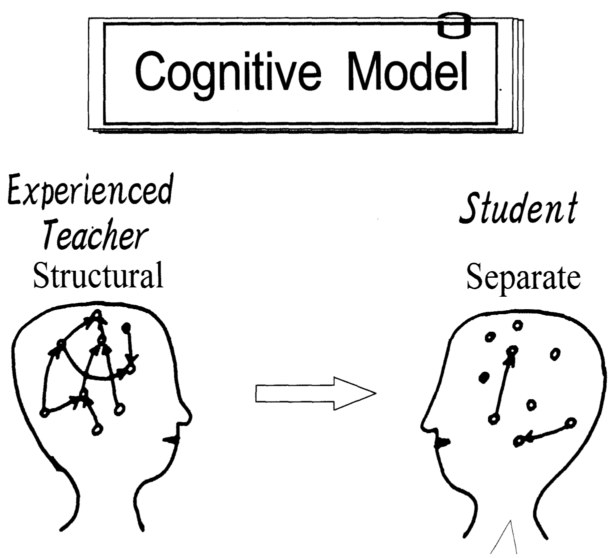


Fig 2

It is assumed that veteran teachers have a structured and systematic grasp and understanding of the subjects and material they teach in class through their teaching experience and previous study of the textbooks, teaching manuals and reference books, i.e., they have a functional network of their knowledge and information formed in their mind by virtue of an understanding of the functional aspects of that knowledge and information.

On the other hand, students may not have a sufficient grasp and understanding of the structural and systematic relationships that exist in subject matter to be learnt as in being presented with new subject matter, their understanding may be limited to individual subjects, or this understanding of each subject itself may be insufficient. In other words, a sufficient functional network of knowledge and information may not have formed in students' minds.

Students can solve problems within each subject matter, but cannot easily solve comprehensive or applied problems that require consolidated or integrated academic abilities if the knowledge structures are not connected to each other in a systematic way in their mind. Moreover, they will not have any interest in the subject matter. Students who can solve exercise problems in a textbook quite well, but who cannot quite solve comprehensive or applied problems in an academic ability test, generally have such a knowledge structure. It goes without saying that it is difficult for these students to show creativity.

In short, a functional network of knowledge and information enables a student to use necessary pieces of knowledge by extracting them one after another from their mind when they solve a problem. Conversely, without such a network of knowledge and information a student will not be able to do so.[6)]

In order to form a functional network of knowledge and information, one must grasp and understand subject matter in a structured and systematic way. It is important, toward this end, to sufficiently assimilate basic and fundamental subjects and material which will form the basis of such a network.

"Basic and fundamental subject matter" does not refer to the quantity of knowledge, but rather to the essential matter that constitutes the basic set of tools needed to acquire mathematical concepts.

"Understanding of basic and fundamental subject matter" refers to both sufficiently understanding this basic set of tools and the ability to understand basic subject matter from a functional perspective and to apply them in various practical settings. In a broad sense, this includes interest in the subject and an incentive to learn.

(2) The current status of the structured and systematic way of thinking

Learning items and keywords which constitute the contents of the units and sections which students learn will be referred to as "learning elements" hereinafter.

Incidentally, to what extent do our students grasp and understand the structural and systematic relationships of learning materials? We have investigated its current status in the following way. First, we gave some of the learning elements that constitute a relevant unit to teachers and students and had them draw a conceptual schematic which uses arrows to show the relationship among the elements. Next, we compared the schematics drawn by the teachers and students and expressed how similar they were via a knowledge transfer index[7)] which measures how much information a teacher has about the relationship existing in the conceptual structure of the relevant teaching unit is transferred to students in the process of teaching-learning. Figure 3 (a) - (d) are typical examples of schematics drawn by a lower secondary school student, a college student, a young teacher and a veteran teacher for the mathematics unit "3 linear functions" taught in the eighth grade. A young teacher has an average teaching experience of approximately ten years.

Lower secondary school students only partially understand the relationship among the learning elements and their grasp remains disparate. Higher secondary school students understand to a considerable degree which element comes immediately ahead of or behind another element, but the overall relationship seems to elude them. Young teachers likewise understand which element comes immediately ahead of or behind another to a considerable degree, but their grasp of the overall relationship appears a little weak. Veteran teachers understand the totality in a structured and systematic way.

Table 2 shows the results of an investigation into what extent elementary school students, lower secondary students, higher secondary students, college students and young teachers understand elementary, lower secondary, higher secondary and university teaching materials[8)]. For instance, the knowledge transfer index is 0.11, 0.20, 0.22 for lower secondary school students, college students and young teachers, respectively, for the mathematics unit “linear functions” taught in the eighth grade, or 2, 3 and 3, respectively, if converted into a 5-step grade. It also ranges from 2 - 3 for materials taught at other kinds of schools or in other grades.

This situation indicates that it would be rather unreasonable to expect students to solve complex problems, to find problems to solve on their own or to demonstrate creativity.

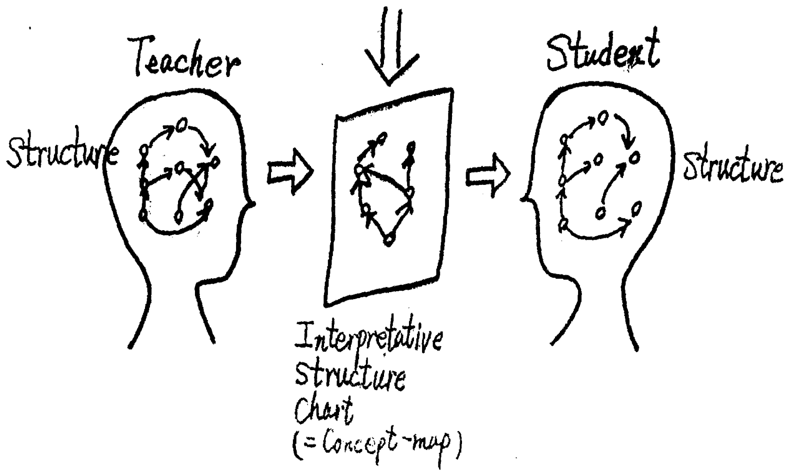
Table 3 shows the knowledge transfer index in cases where training was provided to think in a structured and systematic way and in cases where it was not provided. The 5-step grade rises to 3 - 4 if a schematic was drawn after several sessions of group discussion or training which suggests that although they are not sufficiently used to thinking in a structured and systematic way in their usual classroom activities, such a way of thinking can be instilled through training. This leads to the conclusion that our education until now has not been geared to teaching how to think in a structured and systematic way when studying various subject matter. I hope we have made clear how important it is to instill such a way of thinking in order to build a basis of creativity by assimilating basic and fundamental subject matter.

(3) The aims of the Mountain-climbing learning method

We have shown how important it is to form a functional network of knowledge and information in students’ minds by instilling a structured and systematic way of thinking in order to assimilate basic and fundamental subject matter, and to cultivate a basis of creativity. But how are we to achieve this in practical terms?

One way is to construct a chart that makes visible the structure of concepts that a teacher has in his/her mind and to give that chart to students to use as a learning material. This will accordingly be called a “learning structure chart” or “concept map” [9] as students it use as a learning material.

Figure 4 shows how a teacher can instill a structured way of thinking in his students by externalizing the conceptual structures the teacher has in his/her mind in the form of a learning structure chart and by having his/her students make use of it.



The Mountain-climbing learning method is a technique designed to instill a structured and systematic way of thinking in students through the use of a learning structure chart. The techniques used in practice will be described in the next section, but it is important, for the purpose of instilling this way of thinking,

1) to have students understand the contents of individual learning elements to a satisfactory degree,
2) to have them grasp and understand the relationship among the learning elements, and
3) to have them grasp and understand the relationship between a part and the entirety of the learning contents so that they may see the overall structure of those contents.

Through the above activities, the Mountain-climbing learning method aims to achieve the following:

1) The sufficient assimilation of basic and fundamental subject matter
2) The creation of a functional network of knowledge and information
3) The cultivation of a basis of creativity

2. How to put the Mountain-climbing learning method into practice

We will now make clear in what sequence to put this method into practice, how to use the method for students with different ability levels, and what should be kept in mind when the method is used in teaching.

— Teaching will result in failure if the teacher has no interest or enthusiasm, however good the teaching method may be. —

2.1 How to put the Mountain-climbing learning method into practice

The following sequence is used when the Mountain-climbing learning method is put into practice.[10)]Firstly, we describe how the method is applied to teaching a unit in a textbook.

(1) Preparation by the teacher before the class

The teacher analyzes the contents of the unit material and constructs a “learning structure chart” before the class. Next, he/she makes an “explanatory form of the arrows” and a “self-diagnosis form” with the “learning structure chart” as reference. In total, the teacher prepares three learning materials.

The main material is the “learning structure chart” in the sense that the “explanatory form of the arrows” and the “self-diagnosis form” are made from it and can easily be prepared once it has been completed. A “learning structure chart” is constructed in the following order by a teacher in a practical setting.

(A) Construction of a learning structure chart

Step 1: Extract 10 to 20 learning elements from the learning contents of the entire unit which are the necessary components of the concepts to be introduced, while referring to the textbook or teachers’ manual and drawing on teaching experience and gut feeling. Write the extracted elements down. Here, the elements are usually the learning subjects and key words in the unit.

Figure 5 shows the learning elements extracted from the mathematics unit “3 linear functions” taught in the eighth grade. 13 elements have been extracted.

Step 2: Place the extracted learning elements in their correct position.

In order to do this, examine any two learning elements to see whether they have any relationship such as a presupposive, hierarchical, causal, logical, inclusive, subsumptive or order relationship. Write down the numbers denoting the kinds of relationships found on the right side of each learning element as shown in the subordinate elements field in Figure 5. You may prepare (white) cards of a dimension of approximately 3 cm by 5 cm, writing each learning element on a card and arranging them according to their relationship with one another.

Step 3: Arrange all of the learning elements in a hierarchy

Arrange all of the learning elements in a hierarchy according to whether there is any relationship between them. In the hierarchical arrangement, place subordinate learning elements below superordinate learning elements, superordinate learning elements above subordinate learning elements, and equivalent or dual learning elements on the same level.

The elements are placed from top to bottom, left to right, so it is easily done once you get the hang of it. Learning elements are arranged essentially as in Figure 6 where the symbol ((A), (B), (C), (D)) denotes a learning element.

As for the top to bottom arrangement (1) in Figure 6, we have placed (A) before (B), taking into consideration presupposive or contextual relationships not mentioned in **Step 2** above such as “customs,” “experience,” “subjectivity” and “consensus with others.” As for the paratactic arrangement in (2), we have placed (A) and (B) on the same hierarchical level where their relationships are reciprocal, equivalent or dual.

Figure 7 shows the hierarchical arrangement according to the above procedure for learning elements for the unit “3 linear functions” taught in the eighth grade.

There are two points to be kept in mind when you arrange the learning elements hierarchically in order to make the “learning structure chart” easy for students to view and understand.

The first is that you can make it easier to understand by basically arranging the elements by positioning them on a timeline, i.e., placing an element on the right of the element to be learned previously. This, however, is only a general rule and when the arrows showing the relationships intersect in a complicated way, the order can be reversed to make the chart easy to see as described in the next step, Step 4.

The second point is that elements on the same hierarchical level should be placed on the same horizontal-line position in order to avoid confusing students looking at elements on the same level scattered on different lines.

Step 4: draw in arrows to indicate the relationship between the learning elements.

Figure 7 shows a learning chart with arrows drawn in to show the relationships. Arrows should be drawn in such a way that it is easy to understand what is on the chart, so change the positions of the elements to avoid too much intersecting of arrows as mentioned in Step 3. Also, the arrows should be drawn upward. A downward arrow suggests there is an error in the hierarchical arrangement. The trick is to make all the arrows face upward for an easy view by closely examining the hierarchical arrangement. You cannot expect your students to understand the chart well if it is not easy to see the relationships because of arrows going in various directions.

Step 5: Correct any mistakes by reexamining the arrows of the drawn learning structure chart.

An easy way to check to see if there have been any mistakes made in the hierarchical arrangement of the learning elements or the relationships indicated by the arrows, for example, is to focus attention on any one element, k, and to see if the learning elements on the subordinate or equivalent level that are connected the learning element k are necessary elements in the learning element k. This is a much easier method than going upward from a subordinate element to its superordinate element. If you take a look at learning element 7, “How to write an equation of a linear function, given a slope and an intercept of the graph,” in the learning structure chart shown in Figure 9, for example, by tracing the downward arrows from it, you come to learning element 4, “The graph, intercept, and slope of a linear function,” which gives the basic form of a linear function and defines the relevant terms, and learning element 5, “How to draw a graph, given an intercept and a slope.” Students must have the basic form of a linear function and the definition of the terms used, and must know how to draw a graph of a linear function before they can write the equation of a linear function. Thus, you can deduce that the arrows from learning element 4 to 7, and 5 to 7 are indeed correct.

When all of the learning elements have been examined in this way, you have a completed learning structure chart. Your learning structure chart may become complex while you are getting used to making one by you seeing relationships everywhere, but you can draw a chart that is easy to understand if you first draw an arrow to be a main trunk line and then add the minimum necessary relationships.

The relationship of the learning structure chart indicated by the arrows may vary from teacher to a teacher in its detail, but there will be no problem or worry in using them as long as the main trunk line (relationship) is drawn correctly.

In general, if the arrows show complexity on the learning structure chart for a unit when drawing it, then that unit will have a complex structure of concepts and will therefore be hard for students to understand. On the other hand, if the chart has few arrows and a simple look, then the unit will be easy for students to understand. You can thus have a fair idea whether or not it will be easy for students to understand the learning contents of a unit by drawing a learning structure chart for the unit.

We have explained above how to hand-draw a learning structure chart with only a few learning elements. When there are many learning elements, for instance, more than 30, then the ISM structured learning material software comes in handy. It runs on computer and automatically prints out a learning structure chart which is easy to view and understand. You can produce a learning structure chart that is much more viewer-friendly by using the software when you analyze the structure of an entire textbook (all of the units) or all of the units taught in lower secondary schools (grades 7 to 9) because there are many arrows intersecting each other in a complex way. You can, of course, hand-draw a chart for a whole textbook if the number of elements to be extracted is small.

You may feel it is difficult to construct a "learning structure chart," but once you are used to doing it, you can draw a chart in a relatively short amount of time. It will be difficult for a beginner to construct a chart, but your efforts will be rewarded by your students' improved abilities, higher interest levels and heightened enthusiasm to learn. The procedure can be done by any teacher. Learning structure charts for all units through the 7th to 9th grade are attached at the end of this book as a reference for those of you who would like a sample. You may use the attached charts as a reference if you need to redo your own charts for easier use.

The work of analyzing the structure of the teaching material of the unit from Steps 1 to 5 to be taught and constructing a learning structure chart for the unit has an additional effect of "improving the teacher's ability to analyze teaching material and ability to teach." We have had reports from some teachers that their point of view became broader through the process of constructing a learning structure chart. This shows, as the teachers thought about the contents of the learning elements, the relationships existing between one element and another and the relationship of all the elements as a whole, that their memory capacity grew and their range of thinking broadened as new knowledge and information was added to that already possessed, and their knowledge became more detailed or cohesive.

(B) Making of an "explanatory form of arrows"

A teacher makes an "explanatory form of arrows" for his/her students to fill in the relationships indicated by the arrows in the learning structure chart. Each arrow in the form is represented by the learning element numbers of the learning structure chart, for example: $1 \rightarrow 2$, $2 \rightarrow 4$, etc.

Table 4 shows an “explanatory form of arrows” for the unit “3 linear functions” taught in the eighth grade.

By having students complete an “explanatory form of arrows,” we can expect the following effects:

1) Students will have a more detailed and clearer knowledge of the contents of the individual learning elements.
2) Students will not only have a deeper understanding of the relationships among the learning elements, but also grasp the relationships from their functional aspects and systematize them.
3) Students will be able to organize the learning elements into groups of related and cohesive elements, and therefore increase their memory capacity.

(C) How to make a “self-diagnosis form”

Teachers make a “self-diagnosis form” in the way described below.

Table 5 shows a “self-diagnosis form” for the unit “3 linear functions” taught in the eighth grade. The “learning elements” section in the table has the learning elements corresponding to those on the learning structure chart. Have students review it after the class and indicate the degree of their understanding of the learning elements with symbols such as ◎, ○ and △ indicating “understood very well,” “understood fairly well” and “understood not very well,” respectively. In the section “Questions and things I want to research further,” have students enter questions they had during the class or in their studies or what they wish to study further. Have them cross out the items in this section as they are answered or solved by reviewing them later. By crossing out, students can see what questions they had before and what they thought at the time. They therefore need to be instructed in advance not to use an eraser.

Incidentally, you must keep in mind when you have your students self-diagnose (or self-evaluate), that the aims of this activity for the students are:

- that the self-diagnosis will be a resource (motivating force) to improve themselves, and
- that the self-diagnosis will be a resource (motivating force) to deepen their thoughts.

In asking your students: “Did you understand today’s class material?” or “Was it taught in an effective way?” or “Did you enjoy the class?”, for example, and having them rate the class activity in a 5-step evaluation, is not to have your students self-evaluate, but rather it is an evaluation of the teaching so that the teacher might improve the way he/she teaches. Students’ self-diagnosis aims to encourage student growth by making them self-aware of what their aspirations for the next step of their studies are and what they wish to study further. In other words, it is necessary to encourage student growth in the future rather than judging their abilities of today.

By having students complete the “self-diagnosis form,” we can expect the following effects:

1) Students will have a greater ability to find new problems on their own.
2) Students will have a greater ability to read the contents of learning materials carefully.
3) Students will have greater interest in exploring the contents of materials further.

While students' goals are likely to be focused on understanding the contents of learning materials in the regular class, it is essential for their growth not just to understand those materials in the regular class, but to have the motivation to explore what they have learned further. It is also important for them to develop the habit of finding problems and solutions on their own in regular classroom activities. Methods for making students find problems and solutions on their own will be discussed in Section (2) of the teaching sequence.

Since the self-diagnosis form mentioned in this section describes the subject matter that students themselves have discovered as an object of exploration, we will call it the "exploration card." Although ordinarily a teacher cannot know the subject matter that students have formed in their minds, externalizing the subject matter allows teachers to understand it and makes it easy to follow the changes or transformations in the minds of their students in a time sequence.

The "exploration card" is very effective in helping students find problems on their own and allowing teachers to know the changes in the minds of their students.

That completes the preparation by the teacher before the class.

(2) Teaching sequence

In this section, we will describe the teaching sequence using the three forms—"learning structure chart," "explanatory form of the arrows" and "exploration card"—that are prepared by the teacher prior to the class.

Step 1: At the beginning of the class, the teacher hands out the "learning structure charts," "explanatory forms of the arrows" and "exploration cards" to students.

If the group (class) seems to have a low level of academic ability, only hand out the "learning structure charts." If the academic level of the class is acceptable, standard or above standard, hand out all three forms - the "learning structure charts," "explanatory forms of the arrows" and "exploration cards." A student who does not have a sufficient level of understanding about learning contents is likely to fill in the "exploration card" with impressions of the class, such as "I understood the subject matter" or "I didn't understand it," which do not lead to any worthwhile questions or problems. Since in those cases, using the "exploration card" may cause the student to lose interest in mathematics, there is a need to consider whether to use it or not by carefully examining the current situation of the students. However, since the "exploration card" is effective in detecting students' abilities to find problems and in improving their creativity, it is desirable that teachers use it as much as possible based on careful guidance.

Step 2: The teacher distributes the "learning structure charts" to students, and explains the purpose of using these forms, especially the learning structure chart. For example, the following explanations are provided to the students:

"When you graduate from high school or university and go out into the rapidly changing world in the future, there will be a need to develop the ability to conceive and create valuable products on your own and to explain these products in order to gain the understanding of other people. In other words, you will be required to have the ability to find problems and to communicate information. By making use of this learning structure chart, you will be able to gain a proper understanding of basic and fundamental subject matter and improve your abilities required for the subject. The chart will help you form a structured way of thinking and develop your creativity."

Explanations like this will arouse student interest in the learning structure chart and encourage them to use it of their own accord.

Step 3: The teacher instructs students to enter in the margin of the learning structure chart summaries of what they have learned about individual learning elements. Students write down the following sorts of items in the margin of the learning structure chart:

1) Explanations of the terms they have learned in the class,
2) Mathematical formulas, or
3) Typical exercises or questions and answers they have produced on their own (question making).

In mathematical learning, it is important not just to memorize formulas, but to be able to cite practical examples. We will call “explanations of terms,” “formulas” and “typical exercises (or question making)” the “summary triplet” of learning contents. By making it a rule to summarize the contents of learning using the summary triplet, students are allowed to deepen their understanding of each learning element.

Ask students to write down the sentences used in exercises, problems in the textbook with values changed, or problems they have created on their own for question making.

With students who seem to have rather low academic ability, making them write down typical exercises in the textbook is effective, while with those who have ability at the standard level or higher, it is more effective to make them invent problems on their own.[11)] Inventing problems by oneself helps to deepen understanding of the structure of the problems and to retain that understanding longer.

It is more effective to have students practice the learning structure chart in class at the beginning of an academic year for about 15 to 30 minutes and then to make them work on it for homework. In this way, they are able to think for as long as they need to gain a full understanding by examining materials such as textbooks, notebooks and collections of questions when reviewing the subject matter. And since teachers don’t lose any class hours by using the learning structure chart, they can conduct their classes without worrying about how much of the textbook they have covered in class.

The learning structure chart is produced to help students deepen their understanding of basic and fundamental subject matter and to instill a structured way of thinking by encouraging them to think about the contents and meaning of individual learning elements and the structural relationships among them.

From the viewpoint of cognitive psychology, making students fill in the margin of the learning structure chart with summaries such as explanations of terms, mathematical formulas and examples, means making them rehearse the contents of learning. It also has the effect of promoting the redirection of knowledge and information to commit them to long-term memory.

Figure 8 shows a learning structure chart with a “summary triplet” filled in by an 8th grader. We can see from the chart how the student has reflected on what has been learned from various perspectives and has summarized the contents with ingenuity.

Step 4: Using the learning structure chart as reference, students fill in the reasons for the arrows in the “explanatory form of the arrows” and write down their questions and the problems they wish to explore further on the “exploration card.”

The “explanatory form of the arrows” is drawn to help students clearly understand the contents and meaning of each learning element and to grasp the relationships among the learning elements and between the parts and their entirety by reflecting on the reasons for the arrows, as well as allowing them to organize knowledge and information in a structured and systematic way.

The “exploration card” aims to raise the problem consciousness of students about the subject matter and to improve their ability and motivation to find new and worthwhile questions on their own by making them carefully read the contents in order to gain a more detailed understanding.

To put it simply, the “learning structure chart” aims to make students “summarize learning contents and understand the outline of the concepts of the contents as a whole,” the “explanatory form of the arrows” to help them deepen their “understanding of the relationships,” while the “exploration card” aims to encourage them to “find new problems.” These steps are important as a basis for cultivating creativity.

Table 6 shows an explanatory form of the arrows drawn by a student, and Table 7 is an exploration card filled in by the same student.

These tables allow us to see the internal process through which the student applies his/her thinking to understand the subject matter and the relationship among the learning elements.

Step 5: At the end of a teaching unit when the subject matter is summarized, the teacher again asks students to reexamine the contents of the learning structure chart to present reports and hold discussions on a range of issues, such as the contents and meaning of each learning element, the structural relationships existing in the learning contents as a whole and the outline of those contents. Reexamining the learning structure chart helps students to take notice of the contents, showing that they were missing when the chart was being filled in (acquiring detailed knowledge), and to organize the learning contents as a related whole (acquiring organized knowledge).

Presentations and discussions on learning contents allow students to verbally externalize the conceptual structure of the teaching unit that they have in their mind, and help them acquire a more detailed, contextualized and systematic understanding of the contents to form a functional network of knowledge and information, improving their communicative ability.

Externalizing the thoughts in one’s mind verbally and describing them using written expressions reduces redundancy in knowledge and information, clarifies knowledge and thereby improves the ability to think logically and to express thoughts. Therefore, efforts to externalize thoughts verbally and to describe them logically using written expressions are indispensable for learning.

When you ask students to present a summary report on the structural relationships existing in learning contents as a whole, allow them in the initial stage to look at the learning structure chart when they get lost in their report, as it is difficult to make a good report from the outset (first hour of the class). Then in the second stage, have them report verbally without looking at the chart (second hour of the class). Since presentations and discussions take time, it is also recommended that you get team-teaching support and use these practices in a group of 10 to 15 suited to small-group learning.

Appendix 4
The Five Process Standards proposed by NCTM

Following the five content standards, Principles and Standards lists five process standards:
• Problem Solving
• Reasoning and Proof
• Communication
• Connections
• Representation

The process standards refer to the mathematical process through which students should acquire and use mathematical knowledge. The statement of the five process standards can be found in Table 1.1 (p. 5).
The five process standards should not be regarded as separate content or strands in the mathematics curriculum. Rather, they direct the methods or processes of doing all mathematics and, therefore, should be seen as integral components of all mathematics learning and teaching.
To teach in a way that reflects these process standards is one of the best definitions of what it means to teach "according to the Standards."

Problem Solving
Problem solving is much more than finding answers to word problems and exercises labeled "problem solving." The Problem Solving standard says that all students should "build new mathematical knowledge through problem solving" (NCTM, 2000, p. 52). This statement clearly indicates that problem solving is to be viewed as the vehicle through which children develop mathematical ideas. Learning and doing mathematics as you solve problems is probably the most significant difference in what the Standards indicate and the way you most likely experienced mathematics.

Reasoning and Proof
If problem solving is the focus of mathematics, reasoning is the logical thinking that helps us decide if and why our answers make sense. Students need to develop the habit of providing an argument or a rationale as an integral part of every answer. Children can and do learn that the reasons for their answers are at least as important as the answers themselves. The habit of providing reasons for answers is best started in kindergarten. However, it is never too late to learn the satisfaction and the value of defending ideas through logical argument.

Communication
The Communication standard points to the importance of being able to talk about, write about, describe, and explain mathematical ideas. Learning to communicate in mathematics fosters interaction and exploration of ideas in the classroom as students learn in an active, verbal environment. No better way exists for wrestling with an idea than to attempt to articulate it to others. Mathematical expression, therefore, is part of the process and not an end in itself.

Connections
The Connections standard has two separate thrusts. First, the standard refers to connections within and among mathematical ideas. For example, addition and subtraction are intimately related; fractional parts of a whole are connected to concepts of decimals and percents. Students should be helped to see how mathematical ideas build on one another in useful network of connected ideas. Mathematics is not a laundry list of isolated rules and formulas.
Second, mathematics should be connected to the real world and to other disciplines. Children should see that mathematics plays a significant role in art, science, and social studies. This suggests that

mathematics should frequently be integrated with other discipline areas and that applications of mathematics in the real world should be explored.

Representation

One of the reasons mathematics is so powerful is the way ideas can be expressed with symbols, charts, graphs, and diagrams.

Symbolism in mathematics, along with visual aids such as charts and graphs, should be understood by students as ways of communicating mathematical ideas to other people. Symbols, graphs, and charts, as well as physical representations such as counters or fraction "pie pieces," are also powerful learning tools. Moving from one representation to another is an important way to add understanding to an idea. As teachers help develop flexibility with a variety of representations for mathematical ideas, students not only add to their own understanding but also acquire skill in applying mathematical ideas to new areas and communicating ideas to others.

TABLE 1.1

The Five Process Standards from *Principles and Standards for School Mathematics*

Problem Solving Standard Instructional programs from prekindergarten through grade 12 should enable all students to–	• Build new mathematical knowledge through problem solving • Solve problems that arise in mathematics and in other contexts • Apply and adapt a variety of appropriate strategies to solve problems • Monitor and reflect on the process of mathematical problem solving
Reasoning and Proof Standard Instructional programs from prekindergarten through grade 12 should enable all students to–	• Recognize reasoning and proof as fundamental aspects of mathematics • Make and investigate mathematical conjectures • Develop and evaluate mathematical arguments and proofs • Select and use various types of reasoning and methods of proof
Communication Standard Instructional programs from prekindergarten through grade 12 should enable all students to–	• Organize and consolidate their mathematical thinking through communication • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others • Analyze and evaluate the mathematical thinking and strategies of others • Use the language of mathematics to express mathematical ideas precisely
Connections Standard Instructional programs from prekindergarten through grade 12 should enable all students to–	• Recognize and use connections among mathematical ideas • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole • Recognize and apply mathematics in contexts outside of mathematics
Representation Standard Instructional programs from prekindergarten through grade 12 should enable all students to–	• Create and use representations to organize, record, and communicate mathematical ideas • Select, apply, and translate among mathematical representations to solve problems • Use representations to model and interpret physical, social, and mathematical phenomena

Source: Reprinted with permission from *Principles and Standards for School Mathematics.*

Appendix 5
Developing Effective Use of the Blackboard through Lesson Study

Makoto Yoshida

In Japan, carefully planned and well-organized blackboard use during a lesson is considered one of the most important teaching skills that fosters student understanding. In this paper, Makoto Yoshida, an expert in lesson study and Japanese teaching practices, describes how Japanese teachers develop effective use of the blackboard. He explains a typical lesson process in Japan and how the use of the blackboard fits into that process. Yoshida describes six important functions of blackboard use in Japan: (1) keeping a record of the lesson, (2) helping students remember what they need to do and think about, (3) helping students see the connection between different parts of the lesson and the progression of the lesson, (4) contrasting and discussing ideas students present, (5) helping to organize student thinking and discover new ideas, and (6) fostering organized student note taking skills by modeling logical organization.

Use of the Blackboard and Lesson Study

In Japan, carefully planned and well-organized blackboard use during a lesson is considered one of the most important teaching skills that fosters student understanding of the topic they learn during a lesson. Japanese teachers generally believe that well-organized and coherently presented blackboard writing helps students see the progress of the lesson as well as how the class discussed the various solutions and how the class as a whole reached the conclusion of the lesson. In addition, it is said that a well-organized blackboard helps students organize their thinking and learn to organize their notes (Yanase 1990). For these reasons, Japanese teachers often discuss how they use or organize the blackboard when they develop a research lesson through lesson study.

When we teach a lesson, how often do we think carefully about how we organize the blackboard so students can organize their thoughts, see the connections, understand the material being studied, and organize their notes better? Professional development models that provide collaborative discussion based on observation of actual teaching in classrooms, such as lesson study, can help teachers think about this type of detailed teaching practice that enhances student thinking and understanding.

What Does Research Say about Use of the Blackboard in Japan?

Before I describe how Japanese teachers use or organize the blackboard, I would like to share some interesting research related to the topic of blackboard use. The 1995 Third International Mathematics and Science Study (TIMSS) Videotape Classroom Study (Stigler, et al., 1999) investigated the percentage of use of the chalkboard versus overhead projector in classrooms in three countries: the United States, Germany, and Japan (See Figure 1).

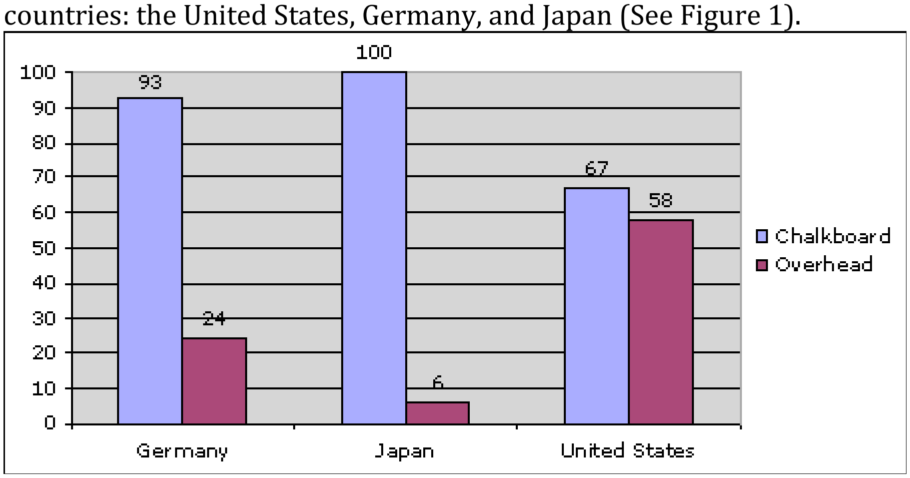
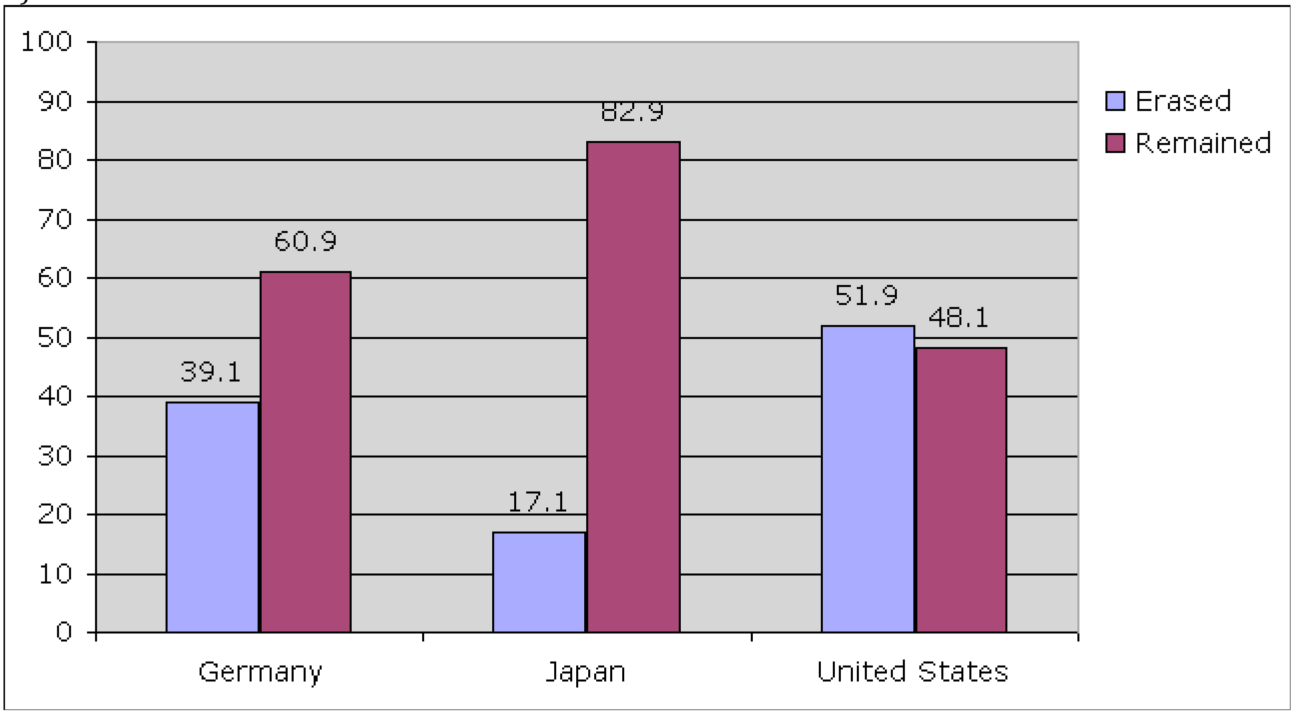


Figure 1: Percentage of lessons in which chalkboard and overhead projector are used. (Source: U.S. Department of Education, National Center for Education Statistics. Third International Mathematics and Science Study, Videotape Classroom Study, 1994-95.)

According to the study, Japanese teachers use the chalkboard all the time and rarely use an overhead projector in the classroom. On the other hand, American teachers use the chalkboard and the overhead projector almost equally. As can be seen in the graph below, TIMSS also reported that Japanese teachers tend to keep what they write on the blackboard until the end of the lesson (See Figure 2).



In the case of American classrooms, over half of what is written on the blackboard is erased by the end of the lesson.

Figure 2: Percentage of tasks, situations, and PPDs (principals/properties/definitions) written on the chalkboard that were erased or remained on the chalkboard at the end of the lesson. (Source: U.S.Department of Education, National Center for Education Statistics. Third International Mathematics and Science Study, Videotape Classroom Study, 1994-95.)

What do these graphs tell us? Why are there differences in the way the two groups of teachers use the blackboard? The researchers who conducted the Videotape Classroom Study, Stigler and Hiebert (1999), talk about it in their book entitled *The Teaching Gap*. First of all, they conclude that the typical use of an overhead projector and blackboard differs between the U.S. and Japan. In the U.S., the blackboard or overhead projector is used for one of the following: 1) to focus students' attention and 2) to display information in written or graphic form. In Japan, however, teachers use the blackboard to provide a record of the problems, solution methods, and principles that are discussed during the lesson.

I (Yoshida 1999) report similar characteristics of Japanese teachers' use of blackboard in the classroom in my ethnographical research on lesson study in Japan entitled *Lesson Study: A Case Study of a Japanese Approach to Improving Instruction Through School-Based Teacher Development*. I describe that the "Japanese teachers rarely erase what they write on the blackboard. Everything they choose to record has a meaning and purpose, as it has been carefully planned in advance." In addition, I record an interview with a teacher who talked about what she was told by senior teachers about use of the blackboard. This teacher told me that "my senior teachers told me 'you should not erase what you write if you write on the blackboard and you should not write on the board if you are going to erase it." Another teacher said, "I try to organize the blackboard in such a way that my students and I can see and understand how the lesson progressed, what was talked about during the lesson and at the end of the lesson."

Lesson Coherence

Stigler and Hiebert (1999) talk about the importance of the connectedness of mathematics across the lesson and how vital it is in helping students develop a clear understanding of the topic taught in the lesson. They call this "lesson coherence." They liken a coherent lesson to a well-formed story that helps students make sense of what is going on in the lesson. A well formed story, according to Stigler and Hiebert, is a sequence of events that fit together to reach

the final conclusion, making it easier to comprehend than a disjointed story.
Becker, et al. (1990) and Stevenson and Stigler (1992) also write about the lesson processes that Japanese teachers follow during mathematics lessons. About half of the open-ended Japanese lessons start with the teacher posing a rich problem, followed by the students struggling with the problem on their own. Next, the students typically present their ideas for solutions and discuss them. Finally the teacher concludes the lesson. This lesson process pattern is also reflected in the lesson plan that the Japanese teachers develop during lesson study.

The open-ended lesson process follows this sequence:

1. introduction to the problem
2. understanding and solving the problem
3. development (includes presentation of solutions by students and comparing and discussing the solutions
4. conclusion or summary of the lesson.

When Japanese teachers follow this process and plan a lesson, they spend a lot of time thinking about:

1. what is a focused mathematical idea
2. how each section of the lesson can be coherently organized in order to develop students' conceptual understanding of the concept taught
3. how they can facilitate the discussion of the student ideas being presented
4. how they can build on the things the class discussed to lead students to understanding.

What teachers focus on when planning a lesson is also reflected in the way they use the blackboard to enhance student understanding. Blackboard planning is called Bansho-keikaku in Japanese. Japanese teachers strongly believe that a well-organized lesson plan and blackboard plan lead to a well-constructed and focused lesson, which in turn helps student understanding.
Following are an example of blackboard planning and an actual picture of a blackboard from a Japanese lesson that I observed. In this lesson, five teachers from the first and second grades in Hiroshima, Japan collaboratively planned a lesson on simple subtraction that involves regrouping.
Figure 3 is the blackboard plan that was created along with the lesson plan.
Figure 4 is a picture that was taken during the actual lesson. Although the teacher digressed from the blackboard plan slightly during the actual lesson, the general plan detailed in the blackboard plan was used during the lesson. When the lesson was completed, the blackboard provided a visual summary of the concepts learned during the lesson.

Figure 3: Blackboard plan the teacher discussed before implementing a research lesson.

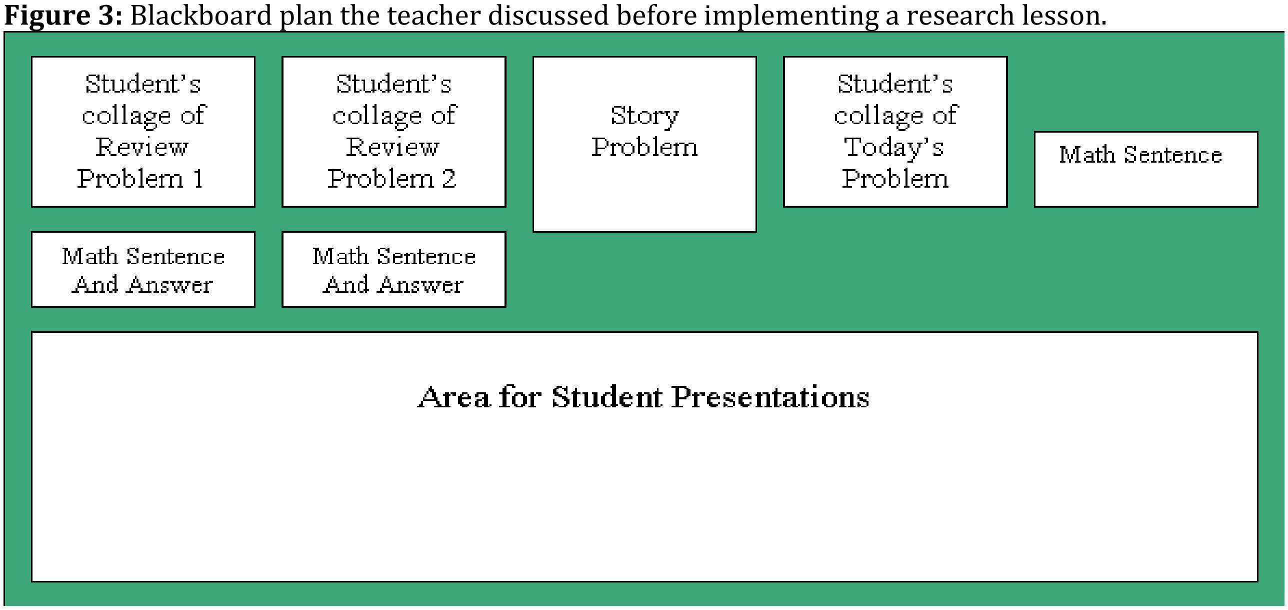


Figure 4: A snapshot of the blackboard from an actual lesson.

How Do Japanese Teachers Use the Blackboard?

It is clear from the description above that Japanese teachers think about and use the blackboard very differently than their American counterparts. It is important to examine closely how Japanese teachers use the blackboard during their lessons. Following are six important functions of blackboard use in Japan.

Keeping a Record of the Lesson

As TIMSS research indicated (Stigler, et al., 1999) the blackboard is used for keeping a record of the lesson. These would be the actual story problem the class worked on during the lesson; the main questions that provoked student thinking; student voices, opinions, and things noticed; various solutions determined by the students; questions and decisions resulting from student discussions; and important mathematical ideas generated by discussions. Keeping a record of the lesson is very useful when the teacher wants to refer to something that happened or was discussed earlier in the lesson. By looking at the blackboard during and after the lesson, students can gain a lot of information, which will go a long way to help them make sense of what they are learning.

Helping Students Remember What They Need to Do and Think About

Keeping the story problem, directions, tasks, and questions on the blackboard provides a place for students to check what they are supposed to do. If students forget what to do or what to answer while they are engaged in the learning activity, they can simply look at the blackboard to obtain necessary information to get back on track. Students can also refer each other to what is on the blackboard to help each other.

Helping Students See the Connection Between Different Parts of the Lesson and the Progression of the Lesson

A well-organized blackboard documents coherence of the lesson. At the end of the lesson, it shows a coherent flow of the lesson, which in turn helps students to see the logical connections among all parts of the lesson. It also shows how the lesson progressed, how student ideas were incorporated into the lesson, and how the conclusion of the lesson was reached.

Contrasting and Discussing Ideas Students Present

Since various student ideas are presented on the blackboard during Japanese mathematics lessons, it becomes a place for students to discuss the presented ideas. The presented ideas are discussed and the similarities and differences in ideas are determined. In addition, the merits of using a certain method to solve the problems are also discussed. Through those discussions, the students might develop new ideas or questions they want to investigate. I call this type of blackboard use a "collective think-pad" because a whole class discussion is carried out based on the ideas presented on the blackboard.

Helping to Organize Student Thinking and Discover New Ideas

The blackboard can also be used for manipulating presented materials to help organize student thinking and discover new ideas. I also call this type of use a "collective think-pad." For example, sorting, lining up, categorizing, and moving directions can be helpful for students to think about, discover, and discuss new ideas. Teachers can facilitate the discussion and help them think about important mathematical ideas. Such use of the blackboard, combined with the discussion on the similarity and differences of the presented students' ideas, is critical for Japanese teachers to skillfully carry out child-centered or discovery-oriented lessons.

Fostering Organized Student Note Taking Skills by Modeling Good Organization

The way teachers organize the blackboard can also be a model for students to take notes during the lesson. Students do not intuitively have good note taking skills, so having a good example to learn from is very important. In this way students can see what is considered good note taking.

Conclusion

In the age of integration of technology in the classroom, many people may think that the blackboard is an old-fashioned instructional tool that has no impact on student learning. However, the innovative use of the blackboard, as is common in Japan, can have a profound effect on student learning in the classroom. Moreover, professional development methods like lesson study can provide important opportunities for teachers to explore new and effective ways to use the blackboard to enhance student thinking and understanding.

References

Becker, J. P., Silver, E. A., Kantowski, M. G., Travers, K. J., & Wilson, J. W. (1990). Some Observations of Mathematics Teaching in Japanese Elementary and Junior High Schools. *Arithmetic Teacher*, 38, 12–21.

Stevenson, H. W., & Stigler, J. W. (1992). *The Learning Gap: Why Our Schools Are Failing and What We Can Learn From Japanese and Chinese Education*. New York, NY: Summit Books.

Stigler, J.W., Gonzales, P.A., Kawanka, T., Knoll, S., & Serrano S. (1999). *The TIMSS Videotape Classroom Study: Methods and Findings from an Exploratory Research Project on Eighth-Grade Mathematics Instruction in Germany, Japan, and the United States*. Washington, DC: U.S. Department of Education.

Stigler, J. W., & Hiebert, J. (1999). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*. New York, NY: The Free Press.

Yanase, O. (1990). *Sansu—Tanoshii Bansho no Giho* [Mathematics—Techniques for Enjoyable Blackboard Use]. Tokyo: Nihonshoseki.

Yoshida, M. (1999). *Lesson Study: A Case Study of a Japanese Approach to Improving Instruction Through School-Based Teacher Development*. Unpublished doctoral dissertation, The University of Chicago.

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Appendix 6
Routine and non-routine problem solving

We can categorize problem solving into two basic types: routine and non-routine. The purposes and the strategies used for solving problems are different for each type.

Routine problem solving

From the curricular point of view, routine problem solving involves using at least one of the four arithmetic operations and/or ratio to solve problems that are practical in nature. Routine problem solving concerns to a large degree the kind of problem solving that serves a socially useful function that has immediate and future payoff. Children typically do routine problem solving as early as age 5 or 6. They combine and separate things such as toys in the course of their normal activities. Adults are regularly called upon to do simple and complex routine problem solving. Here is an example.

> *A sales promotion in a store advertises a TV screen regularly priced at Vt12598 but now selling for 20% discount of the regular price.. You have Vt10000 in your pocket. Do you have enough money to buy the jacket?*

As adults, and as children, we normally want to solve certain kinds of problems (such as the one above) in a way that reflects an '*Aha, I know what is going on here and this is what I need to do to figure out the answer.*' reaction to the problem. We do not want to guess and check or think backwards or make use of similar strategies. Invariably, solving such problems involves using at least one of the four arithmetic operations (and/or ratio). Being good at doing arithmetic (e. g. adding two numbers: mentally, by pencil and paper, with manipulatives, by punching numbers in a calculator) does not guarantee success at solving routine problems. The critical matter is knowing what arithmetic to do in the first place. Actually doing the arithmetic is secondary to the matter.

A mathematics researcher interviewed children about how they solve routine problems. One boy reported his method as follows: *If there were two numbers and they were both big, he subtracted. If there was one large and one small number, he divided. If it did not come out even, he multiplied*. The other interesting aspect of all of this is that the child had done quite well at solving routine problems throughout his school career. What does this say about teaching practice? What does this say about assessing what children understand?

Is the case of the boy an isolated incident or is it the norm? Unfortunately, research tells us that it is likely the norm. Not enough students and adults are good at solving routine problems. Research also tells us that in order for students to be good at routine problem solving they need to learn the meanings of the arithmetic operations (and the concept of ratio) well and in ways that are based on real and familiar experiences. While there are only four arithmetic operations, there are more than four distinct meanings that can be attached to the operations. For example, division has only one meaning: splitting up into equal groups. Subtraction, on the other hand, has at least two meanings: taking away something away from one set or comparing two sets (refer to The meanings of the arithmetic operations.)

Once students understand the meaning of an arithmetic operation they have a powerful conceptual tool to apply to solving routine problems. The primary strategy becomes deciding on what arithmetic operation to use. That decision cannot be made in the manner done by the boy of the research anecdote. The decision should be made on the basis of IDENTIFYING WHAT IS GOING ON IN THE PROBLEM. This approach requires understanding the meanings of the arithmetic operations.

The research evidence suggests that good routine problem solvers have a repertoire of automatic symbol-based and context-based responses to problem situations. They do not rely on manipulating concrete materials, nor on using strategies such as 'guess and check' or 'think backwards'. Rather, they rely on representing what is going on in a problem by selecting from a limited set of mathematical templates or models. Refer to Using arithmetic operation meanings to solve routine problems for details.

Solving routine problems should at some point involve solving complex problems. Complexity can be achieved through multi-step problems (making use of more than one arithmetic operation) or through Fermi problems. It is advisable to do both.

Fermi problems are special problems that are characterized by the need to estimate something and the need to obtain relevant data. They typically involve the application of the meaning of at least one arithmetic operation and sometimes something else (e. g. how to calculate the area of a triangle). Here is an example of a Fermi problem: *About how many cars are there in Manitoba?* Solving this Fermi problem about the cars would involve matters like obtaining/estimating data about the population of Manitoba that might own a car and making use of the ‘groups of’ meaning of multiplication. It could involve more matters. That would depend on the degree of sophistication of insight into the problem.

In general, solving Fermi problems involves estimating where the exact value is often unknown, and perhaps it is even unknowable. While the estimate may be considerably in error, the important matter is on describing how the estimate was obtained. That requires students to justify their reasoning in terms of the meanings of arithmetic operations and in terms of the relevance of the data they collected/estimated.

Non-routine problem solving

Non-routine problem solving serves a different purpose than routine problem solving. While routine problem solving concerns solving problems that are useful for daily living (in the present or in the future), non-routine problem solving concerns that only indirectly. Non-routine problem solving is mostly concerned with developing students’ mathematical reasoning power and fostering the understanding that mathematics is a creative endeavor. From the point of view of students, non-routine problem solving can be challenging and interesting. From the point of view of planning classroom instruction, teachers can use non-routine problem solving to introduce ideas (EXPLORATORY stage of teaching); to deepen and extend understandings of algorithms, skills, and concepts (MAINTENANCE stage of teaching); and to motivate and challenge students (EXPLORATORY and MAINTENANCE stages of teaching). There are other uses as well. Having students do non-routine problem solving can encourage the move from specific to general thinking; in other words, encourage the ability to think in more abstract ways. From the point of view of students growing to adulthood, that ability is becoming more important in today’s technological, complex, and demanding world.

Non-routine problem solving can be seen as evoking an ‘*I tried this and I tried that, and eureka, I finally figured it out.*’ reaction. That involves a search for heuristics (strategies seeking to discover). There is no convenient model or solution path that is readily available to apply to solving a problem. That is in sharp contrast to routine problem solving where there are readily identifiable models (the meanings of the arithmetic operations and the associated templates) to apply to problem situations.

The following is an example of a problem that concerns non-routine problem solving.

> *Consider what happens when 35 is multiplied by 41. The result is 1435. Notice that all four digits of the two multipliers reappear in the product of 1435 (but they are rearranged). One could call numbers such as 35 and 41 as pairs of stubborn numbers because their digits reappear in the product when the two numbers are multiplied together. Find as many pairs of 2-digit stubborn numbers as you can. There are 6 pairs in all (not including 35 & 41).*

Solving problems like the one above normally requires a search for a strategy that seeks to discover a solution (a heuristic). There are many strategies that can be used for solving unfamiliar or unusual problems. The strategies suggested below are teachable to the extent that teachers can encourage and help students to identify, to understand, and to use them. However, non-routine problem solving cannot be approached in an automatized way as can routine problem solving. To say that another way, we cannot find nice, tidy methods of solution for all problems. Inevitably, we will be confronted with a situation that evokes the response; “*I haven't got much of a clue how to do this; let me see what I can try.*”

The list below does not contain strategies like: ‘read the question carefully’, ‘draw a diagram’, or ‘make a table’. Those kinds of strategies are not the essence of what it takes to be successful at non-routine problem solving. They are only preliminary steps that help in getting organized. The hard part still remains - to actually solve the problem - and that takes more powerful strategies than drawing a diagram, reading the question carefully, or making a table. The following list of strategies is appropriate for Early and Middle Years students in that the strategies involve ways of thinking that are likely to be comfortable for these students.

- Look for a pattern
- Guess and check
- Make and solve a simpler problem.
- Work backwards.
- Act it out/make a model.
- Break up the problem into smaller ones and try to solve these first.

It is important that students share how they solved problems so that their classmates are exposed to a variety of strategies as well as the idea that there may be more than one way to reach a solution. It is unwise to force students to use one particular strategy for two important reasons. First, often more than one strategy can be applied to solving a problem. Second, the goal is for students to search for and apply useful strategies, not to train students to make use of a particular strategy.

Finally, non-routine problem solving should not be reserved for special students such as those who finish the regular work early. All students should participate in and be encouraged to succeed at non-routine problem solving. All students can benefit from the kinds of thinking that is involved in non-routine problem solving.

Comparing routine and non-routine problem solving

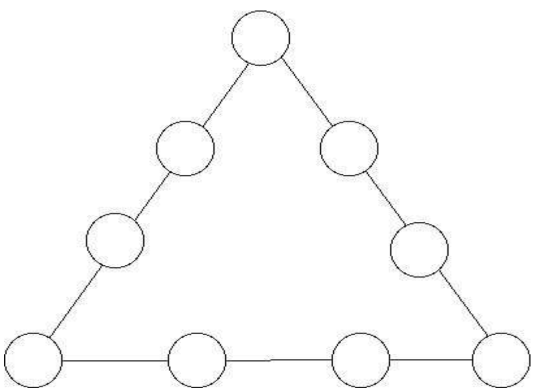
To make clearer the distinction between routine and non-routine problem solving, consider the following two problems. Both are suitable for grade 3.

Problem 1

> *My mom gave me Vt35. My father gave me Vt45. My grandmother gave me Vt85 . How much vatu do I have now?*

Problem 2

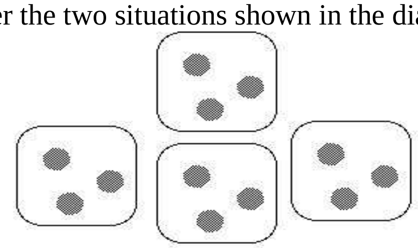
Place the numbers 1 to 9, one in each circle so that the sum of the four numbers along any of the three sides of the triangle is 20. There are 9 circles and 9 numbers to place in the circles. Each circle must have a different number in it.

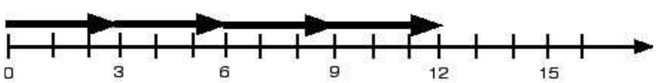


Appendix 7
The meanings of the arithmetic operations (needed for routine problems solving)

The arithmetic operations are mathematical models (symbolic representations/notational systems/sign systems) of certain situations. A model is a way of representing some feature of reality. It is a way of looking at something. **Models are not true or false**. **They are simply useful or not useful for some purpose**. We tend to view the world through models in order to make sense of it. The models we use provide us with lenses through which human behaviour and other phenomena are organized, examined, and evaluated. For example, a set of moral principles is a model of how human beings are supposed to behave. There are numerous models that pertain to teaching. Piaget's model of child development is one example.

Consider the two situations shown in the diagram from the point of view of modeling them mathematically.





Each situation can be viewed as four groups with three in each group. This can be represented or modelled by the number sentence: '4 x 3 = 12'.

Note that, for 4 x 3 = 12, '12' is not the answer to anything because there is no question. The matter is not about questions and answers. It is about mathematically modeling or representing something. If one wishes to view the number sentence '4 x 3 = 12' from the point of view of answers, then there are three "answers". '4' is the answer to how many groups; '3' is the answer to how many objects in each group; and '12' is the answer to how many objects in all. Each number must be "figured out" before it can be placed in its appropriate place in the number sentence.

The four arithmetic operations can be organized into three themes:

- combining,
- separating,
- comparing.

All the meanings relevant to the early years mathematics curricula are indicated here. These meanings are important to developing effective routine problem-solving skills.

COMBINING:

1. put together (+)
2. groups of (x)

COMPARING:

1. two sets (-)

SEPARATING:

1. take away (-)
2. splitting into equal groups (÷)

COMBINING:

(1) The 'put together' meaning of addition

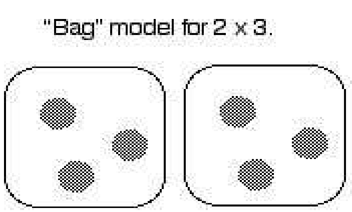
The arithmetic operation of addition has only one meaning – put together. This meaning is represented by a number sentence such as '3 + 5 = 8'. Each symbol in the number sentence has a particular role. '3' is the count of what we already have in the "pot" . '+' indicates that we bring additional stuff into the "pot". '5' is the count of the additional stuff. '=' indicates an action of counting up what we now have in the "pot". '8' is the count of what is in the "pot" after the 'put together' action is over. A pictorial representation of this 'put together' action is shown here.

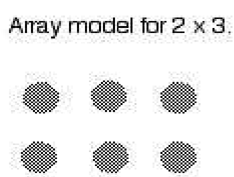
(2) The 'groups of' meaning of multiplication

The 'groups of' meaning of multiplication concerns an **existing arrangement** of equal groups of objects. The multiplication symbol ('X') models the static situation that exists **after equal groups have been formed** rather than on the action of actually forming equal groups. The action of forming the groups is separating/splitting up and that is modeled by division.

There is a convention attached to the 'groups of' meaning of multiplication. For example, in '5 x 3', the first number '5' represents the number of groups; the second number '3' represents the number in each group. Using the convention emphasizes meaning in terms of groups and how many in each group. Other conventions such as 'write the bigger number first' do not emphasize meaning.

Two basic pictorial models are used for the 'groups' of meaning of multiplication: a "bag" model and an array model. The following illustrates a picture of 2 x 3 for each of these models. For the array model, the convention is that the first number is the number of rows and the second number is the number of columns (think spreadsheet).

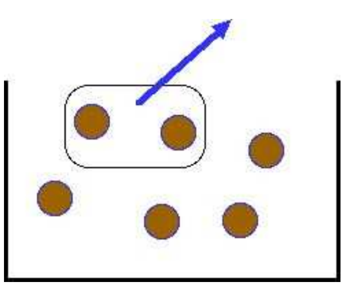




SEPARATING:

(1) The 'take away' meaning of subtraction

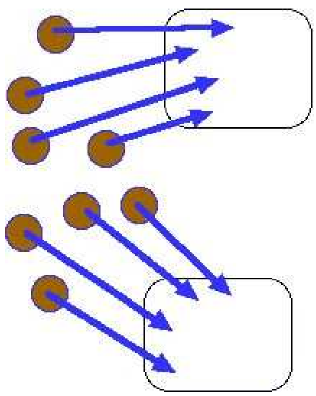
The 'take away' meaning is the fundamental meaning of subtraction. It involves an action of removing things away from a "pot" and counting up what is left. The 'take away' meaning is the one that is directly related to addition. Here is a picture of **6 – 2 = 4,** from the point of view of the 'take away' meaning.



There are some points to keep in mind when teaching the 'take away' meaning of subtraction. Language is important. It is better to use the generic term 'subtract' rather than the context-specific term 'take away' when referring to the arithmetic operation of subtraction. However, it is appropriate to use the term 'take away' when referring to the actual action of taking away. This means that, for number sentences such as '7 - 3 = 4', it is preferable to say "7 subtract 3 is/equals 4" rather than "7 take away 3 is/equals 4". Using the term 'take away' to refer to the operation of subtraction may hinder the development of 'routine problem-solving skills because real world subtraction problems do not always involve the 'take away' meaning (sometimes they involve the comparison meaning of subtraction). Another inadvisable word to use when referring to subtraction is 'minus'. Students are likely aware of that word in relation to temperature. By using the word 'minus' to mean 'subtract', you can create confusion for students now and in later years (when they learn about integers).

(2) The 'splitting up into equal groups' meaning of division

There is only one meaning of division. It represents an action that involves splitting up a collection of objects into equal groups. Here is a picture of **8 ÷ 4 = 2**.



There are a couple of matters to keep in mind when teaching the meaning of division. Students have been interpreting a collection of equal groups in a multiplication way (the 'groups of' meaning). It may not be wise to use the same picture for illustrating multiplication and division. For multiplication, use static pictures. For division, use pictures that convey an action of splitting up into two equal groups (such as the picture above).

There are two very different sorts of questions that one can ask when dividing. One question concerns "*How many groups can I form?*" and the other concerns "*How many are in each group?*" Both questions are normally represented by the same number sentence, for example; '15 ÷ 3 =?'. The '?' can represent either 'the number of groups' or 'the number in each group'. But which is it? Pay attention to both types of questions when teaching students to solve routine problems involving division.

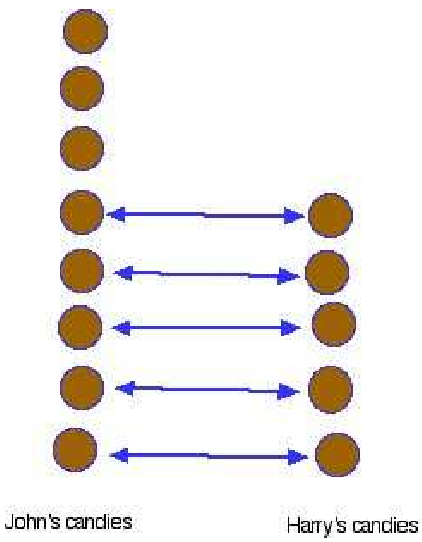
COMPARING:

Comparing involves making decisions about more/less/equal, shorter/taller/same as etc. Comparison can be modeled by subtraction and multiplication. Because comparison by multiplication is a middle years matter, we will only consider comparison by subtraction here. Refer to the comparison meaning of x for a brief consideration of that meaning of multiplication.

The comparison meaning of subtraction

The comparison meaning of subtraction does not involve an action of removing or taking away. It involves comparing two sets as to more/less/same. The heart of the matter for comparing by subtraction is matching the objects of two sets and counting what is unmatched. The unmatched stuff is typically referred to as the difference between the two sets. Consider the following. *John has 8 candies. Harry has 5 candies.* If the candies of the two boys are matched, we will find that there will be 3 candies unmatched. We refer to this situation in many ways: "*The difference between two sets of candies is 3.*" or "*John has 3 more candies than*

Harry." or "*Harry has 3 fewer candies than John.*" In mathematics we can write this as '8 - 5 = 3', a number sentence that looks identical to one that might involve the 'take away' meaning of subtraction. Notice though that there is no removal of stuff when John's and Harry's candies are being compared (no take away action). John still has 8 candies and Harry still has 5. In fact, there are 13 candies in all involved in the situation. Here is a picture of the candies and the matching process.

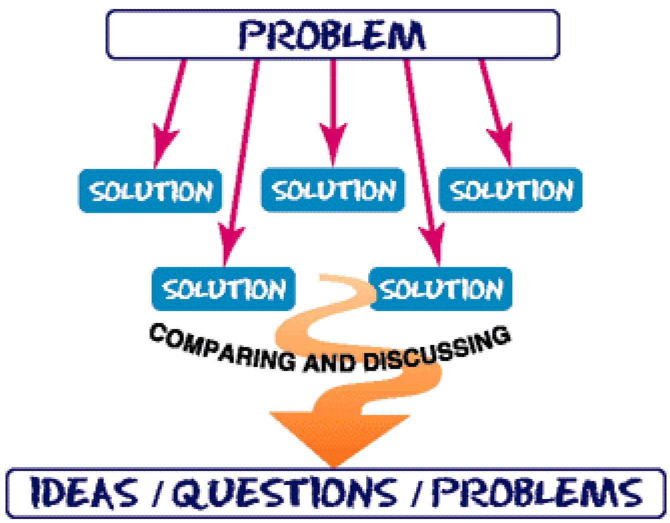


There are some matters to keep in mind when teaching the comparison meaning of subtraction. If students have been taught to say 'take away' when referring to the subtraction symbol, '-', there are likely to be difficulties. They may be confused and even trapped by the faulty language. That is why the generic language, 'subtract', is better language for the mathematical symbol, '-'. The actual actions or situations can be described according to what is happening (losing, comparing, etc.).

Attention needs to be paid to the use of appropriate language such as 'more/less/fewer', 'taller than/shorter than'. In relation to this, we subtract the smaller number from the larger number when using the comparison meaning of subtraction. The mathematical idea of positive/negative is expressed by language such as bigger/smaller. This way negative numbers are not needed.

Appendix 8
What is the Open-Ended Problem Solving?

Open-ended problem is a problem that has several or many correct answers, and several ways to the correct answer(s). The Open-Ended Problem Solving is based on the research conducted by Shimada S., which is called "The Open-Ended Approach". The Open-Ended Approach provides students with "experience in finding something new in the process"(Shimada 1997). The Open-Ended Approach started in 1970s. Since then, Japanese teachers have developed many open-ended problems and lesson plans using open-ended problems. These problems are being used in mathematics lessons elementary through high school grades, and the lessons are called the Open-Ended Problem Solving now. Open-Ended problems are also used as assessment tasks because "In responding to such (open-ended) items, students are often asked not only to show their work, but also to explain how they got their answers or why they chose the method they did" (Schoenfeld, A. et al., 1997). The Open-Ended Problem Solving also has been widely regarded as an advanced style of teaching mathematics in the U.S. recent years.



Advantages of the Open-ended Problem Solving

There are 5 advantages that can be summarized, based on what Sawada mentioned in 1977 (Sawada, 1997),

1). Students participate more actively in lessons and express their Ideas more frequently.

The Open-Ended Problem Solving provides free, responsive, and supportive learning environment because there are many different correct solutions, so that each student has opportunities to get own unique answer(s). Therefore, students are curious about other solutions, and they can compare with and discuss about their solutions each other. As students are very active, it brings a lot of interesting conversation to the classroom.

2). Students have more opportunities to make comprehensive use of their mathematical knowledge and skills.

Since there are many different solutions, students can choose their favorite ways toward the answer(s) and create their unique solution(s). Activities can be the opportunities to make comprehensive use of their mathematical knowledge and skills.

3). Every student can respond to the problem in some significant ways of his / her own.

There are various kinds of students in a mathematics classroom, since there are no tracking in Japanese classroom. Therefore, it is very important for every student to be involved into the classroom activities, and the lessons should be understandable for every student. The open-ended problems provide every student with the opportunities to find his / her own answer(s).

4). The lesson can provide students with a reasoning experience.

Through the comparing and discussing in the classroom, students are intrinsically motivated to give reasons of their solutions to other students. It is a great opportunity for students to develop their mathematical thinking.

5). There are rich experiences for students to have pleasure of discovery and to receive the approval from fellow students.

Since every student has each solution based on each unique thinking, every student is interested in fellow students' solution.

There are also some disadvantages that Sawada mentioned in 1977 (1997), such as difficulty of posing problems successfully, difficulty of developing meaningful problem situations, and difficulty of summarizing the lesson.

Appendix 9
Project-Based Learning in mathematics

Key Questions

Projects result from students' attempts to answer essential questions. They can take many forms: products, presentations, performances. They might fit any of three structures: interpersonal, information sharing, or problem-solving. When selecting an existing project, or creating one of your own, consider the following:

1. Is the project devoted only to mathematics (or a single subject area), or is there a link to other curricular areas?
2. Is the project tied to standards for the curricular areas addressed, such as those from the National Council of Teachers of Mathematics and the National Education Technology Standards?
3. Does the project come with classroom instructional materials (e.g., teacher resources, student activities, rubrics and assessment tools)?
4. Can all students in your class participate? Projects should not be reserved for your talented and gifted students, as all students should be able to benefit.
5. What is the total time for project completion?
6. Is the project collaborative in nature? A collaborative project, particularly involving students outside your own school setting, will take more time and monitoring to help students learn how to be a part of a team and communicate appropriately with others.
7. How will students benefit both academically and personally from their involvement in the project? Consider that when students interact with other students and experts across the country or internationally, they get a broader feel for diversity. Their participation in an actual real world activity might encourage them to do their best work, and see the relevance of mathematics in their daily lives. If students have input into project selection, and like the topic, they will tend to become more involved and excited about their learning.
8. Is there a cost involved to participate?

Appendix 10
PEER TEACHER LEARNING (PTL) OR LESSON STUDY (LS) –

WHAT IS IT AND WHAT PURPOSE DOES IT SERVE?

Lesson Study is an approach used to empower teacher in order to improve their performances in the classroom in order to effectively teach so student learning is improved.

It is currently a topic of world wide attention, It refers to a process in which teachers progressively strive to improve their teaching methods by working together with other teachers to examine and critique one another's teaching techniques. First developed as an educational practice in the Meiji period of Japan, Lesson study functions as a means of enabling teachers to develop and study their own teaching practices. It is this function to which its international attention can be attributed.

This booklet therefore examines the benefits of teachers working together to plan lesson units (Lesson Plans), to carefully observe classroom practice of colleagues and to analyze classroom observation to become more effective teachers.

Therefore, our aim is to provide some suggestions on how to organize, implement and evaluate Peer Teacher Learning (PTL) as part of school based in - service training (INSET) especially in Mathematics.

The introduction of (LS) or (PTL) in this training is to assist teachers to share their knowledge and skills; observe, discuss and evaluate their own classroom practice and others that could result in improved student learning.

It is therefore very practical for schools to develop a culture of improving class teaching through mutual cooperation as currently the national education system has no in - service training program in place to upgrade and continue to improve teacher performance and student learning.

How to organize PTL/LS
PTL should be organized in all schools based on a selected content and on the needs identified by the teachers at the beginning of the academic year. PTL should happen at least 4 times a year. The date(s) and time for this must be decided upon. It is both practical and courteous to inform the principal and obtain permission for the use of the venue and any other school properties or materials.

PTL/LS should involve three stages:
Pre – PTL/LS Conference
PTL/LS Observation, and
Post – PTL/LS Conference

1. Pre – PTL/LS Conference

In this stage teachers meet and discuss the theme of the entire lesson, how does it fit into the entire unit? What are the intended outcomes? Etc…
How are they going to be achieved (Strategies)? When are they going to be achieved? How are they going to be assessed? What sort of resources are going to be used by the learners and by the teacher? Which part of the lesson does the teacher want the observing teachers to concentrate on and give feedback; is it the entire lesson or some specific parts only? They also decide which teacher amongst them will present the lesson on behalf of the group.

The observing teachers should also share with the presenting teacher what they will be doing during the lesson. They should communicate with the teacher whether they will using an instrument (video camera) in observing the lesson or will be taking notes. They should also communicate how they are going to handle the feed back or the reports on the things that they have observed in the lesson. They should always communicate that classroom observation is a learning activity and is not an inspection, and assessment or an evaluation tool to identify good and bad teachers. It is an experimental learning activity for teachers based on the improvement of classroom practices. The focus will and should be on the lesson and not on the teacher. Hammersely (1995 cited in Ndlalane & Nishioka, 2001) emphasized that “teachers monitoring the lesson are directed at improving their understanding; in the process, their criteria for improvement and their views on teaching will change.

The learners that are participating in the actual lesson activity should be made aware that teachers are sharing expertise and experiences through sharing practices and observing each other. The major emphasis should be on supporting the good
Characteristics of what the particular teacher is doing and the rest of them would like to learn from the teacher. The observing teachers should be allocated a place to sit which will not disturb the learners or the teacher that is conducting a lesson.

It will be a good thing for the observing teachers to have a copy of the lesson unit or teaching plan (Lesson Plan). This makes feed back more valuable at the end as feed back will based on something that is tangible. However, it should be borne in mind that feedback is often better when invited rather than imposed.

The key to this is how to improve our lesson unit (Lesson Plan). The teacher who is to teach the lesson should be the facilitator of the pre – PTL/LS Conference so that the lesson remains focused.

1. Select a topic which is difficult to teach or difficult for students to learn;

2. Design the lesson unit (Lesson Plan)

- Prior knowledge of the students
- Content to be covered
- How to sequence the activity
- How it should be taught
- How to assess students’ achievements
- Prepare resources to be used
- Intended outcome of the lesson unit
- Discuss possible ways of improvement;

3. Design an observation tool that provides guidance as to what to attend to in the class (e.g a checklist for lesson analysis)

4. Discuss ways of how to report on the things observed during the lesson.

Pre – Peer Teacher Learning/Lesson Study Conference minutes form

Date: ……/……/….. Place: …………………….. Time: ……………… Facilitator: ……………………………..

Participating teachers:	
Topic Selected for PTL/LS:	
Design of the lesson Unit/Plan:	
Prior Required Knowledge, Skills & Attitudes:	
Content to be covered:	
Intented Outcomes:	
Resources:	
Lesson Activities:	
What to look for (main focus):	
Invitation:	Outside assistance:
Date of observation:	Date for Post – PTL/LS:
Facilitator for Post – PTL/LS:	Report writer for PTL/LS:
Next PTL/LS (PTL/LS 2):	Date for next PTL/LS (PTL/LS 2):
Venue:	Time:
Responsible for organising PTL/LS 2:	

1.1 How to use Lesson Observation Form (Figure 2)

The purpose of this form is to highlight all the events done by the teacher and the learners during the lesson without any interpretation. For example, the teacher might be talking about the differences in how to convert from decimal fraction to a percentage for about 10 minutes as an introduction, while the learners are seated and listening to the teacher. On the form, under "Describe what the teacher is doing." The peers will write 'teacher talking about how to convert decimal fraction to percentage. On the form under"Describe what the learners are doing", the peers will write learners are seated and listening to the teacher for 10 minutes" If the peers have to do the interpretation of this event in form 2, they might interpret this event as a long introduction, and as a teacher – centered approach.

Please remember that this form will be used to give feedback to the teacher. The feedback should mainly focus on the things that you can change or control, not things that cannot be changed, e.g. class size, in giving feedback, one should be very specific and not general. It is a good idea to give examples and quote some of the events that occurred during the lesson. <u>The observing teachers should be</u> <u>descriptive rather than evaluative</u>. Please take notes that would help the presenting teacher understand the situation of the lesson, and that help you
as observing teacher learn lessons yourself and give advice to the presenting teacher.

1.2 How to use Lesson Analysis Checklist (Figure 3)

The purpose of the checklist is to place events into categories as they were described in form 1, but more of an interpretation of an event. **For example, learners were seated in groups talking and discussing the given tasks, while the teacher was moving around from group to group.** This can be interpreted as group work and teacher facilitation for 10 minutes.
Please note that this analysis should be done after the lesson. The observing teachers would better concentrate and taking notes in form 1 during the lesson presentation.

The checklist covers many elements. <u>During the Post PTL/LS conference, it would be more meaningful to decide which elements you would like to improve at the next step.</u>

The checklist can be used also while planning and writing the teaching plan. <u>In this case also, it is important to decide which elements are the main targets of the lesson rather than paying attention to all the elements listed in the checklist at once.</u>

1.3 How to use conference notes

The purpose of this form is to keep records of the discussions of the Post PTL/LS Conference. During the Post PTL/LS Conference, you will discuss the strength and weakness of the lesson, and give feedback to the teacher. The records of such discussion will help all of you to remember the lessons learnt from the PTL/LS.

The conference Notes can be used also for planning the next PTL/LS. For example, if you check what was written in the column of Short term – Targets in the previous PTL/LS, then you will automatically know what should be the main targets of the next PTL/LS.

It is recommended that each school compile the Notes in a folder. It would be a very good record of when you had PTL/LS sessions, what you discussed and improvements recommended and how you have improved. It would also act as the stepping stone for the next PTL/LS.

1.4 Minutes of a Pre – PTL Conference

The minutes of the Pre – PTL/LS Conference will assist in planning and organizing the observation session, the Post – PTL/LS Conference and for future references.

Form Figure 1 below shows a sample completed Pre – PTL/LS form and a vacant form Appendix A.

The format given below can be used in capturing the minutes of Pre – PTL/LS Conference. (Fig 1)

Post – Peer Teacher Learning/Lesson Study Conference minutes form

Date: ……/……/…… Place: …………………… Time: ………………….. Facilitator: …………………………

Participating teachers:	
Topic Selected for PTL/LS:	
Comments by the facilitator: What worked well and why/What did not work well and why.	
Coments by the teacher: What worked well and why/What did not work well and why.	
Comments by the other teachers: What worked well and why/What did not work well and why.	
Comments by teacher on feedback:	
Discuss & compromise on what should be changed:	
Set a short term target:	
Next PTL/LS (PTL/LS 3):	Date for next PTL/LS (PTL/LS 3):
Venue:	Time:
Responsible for organising PTL/LS 3:	

2. PTL/LS Session

During the lesson presentation the observing teachers must describe and document all the things that take place during the lesson as agreed prior to the lesson. As we discussed before, Lesson Observation Form can be used for taking notes while observing. It is important to quote some of the things that the teacher or the learners said or did in the classroom. The detailed experiences should be written down. The observing teachers should not interfere with the lesson presentation of the teacher during the lesson.

The PTL/LS session can be seen as an activity to gather data that can help all teachers to maximize what is learned from the lesson unit.

Below is the list of things the observing teachers may be doing during a PTL/LS session.

- Record the information from classroom activity – Observing teachers, video camera, etc …
- Capture different elements of the lesson unit (teacher activity, student activity, blackboard work, etc …), concentrate on a few aspects.
- Describe and document what the teacher is doing, as agreed prior to the lesson
- Describe what the teacher is doing
- Note the time taken for each activity

2.1 PTL/LS LESSON OBSERVATION FORM (Fig 2)

LESSON STUDY

Peer Teacher Learning/Lesson Study observation form

Date: ……/..…/…… Place: ……………………… Time: ……………… Observer: ………………….....………...

Time	Describe what the teacher is doing.	Describe what the learners are doing.

Eg

School : ………………… Grade : …… Teacher: ……………………………. Date : ………

Time Minutes	**Describe what the teacher is doing.**	**Describe what the learners are doing**
5	**Background and introduction to the lesson. Questions rarely used**	**Mostly listening. Answering prior knowledge questions**
10	**Organizing groups and dividing materials. Explaining instructions. Asking learners to explain to their peers.**	**Listening. Read the statements provided. Learners involved in the explaining questions, that is assisting the teacher.**
15	**Moving around and explaining where necessary. Facilitates the learners' responses**.	**Learners encaged with the activity. Learners give their responses. Wrong responses are corrected by the learners themselves.**
5	**Teacher exposes the concept and summarizes.**	**Learners listening.**
5	**Teacher extends the activity by giving them another activity on folding.**	**Learners listen and then read the given instructions.**
10	**Moves around and explains activity and assess progress of learners.**	**Learners encaged in folding papers and answering questions.**
5	**Teacher exposes the concepts of equivalent ratios and fractions and asks for responses**.	**Learners write the correct answers and answering questions.**
5	**Summarizes lesson**	**Listening**

3. Post – PTL/LS Conference

After the lesson presentation, the teacher who presented the lesson should be given a chance first to reflect on the lesson according to plans or not, e.g. what worked and what did not work and why? The teacher must be given a chance to reveal his/her own position and feelings. This helps to encourage the openness of the teacher and the willingness to learn from the activity. The observing teachers should make sure that they understand the comments from the presenter since this helps when asking and commenting about the matters that have already been discussed by the presenter.

After the comments, the observing teachers should comment on the good aspects of the lesson first then handle the weaknesses and shortcomings of the lesson later. <u>The observing teachers should not focus on the presenter but on the lesson</u>. <u>The observing teachers should avoid, for example, the teacher did not do well but instead say, "the lesson did not come across well.</u>" The observing teachers should come out with suggestions rather than negative criticism of the lesson: for example, "would suggest that instead of doing a, b, c, I suggest that you try d, e, f, g, h and see what difference it is going to make." The focus should be on the activities and not on the teacher. If the instrument was used with the intention of sharing this with the teacher then it will be ideal to discuss the response and performance of the teacher. It is important that as a summary, the observing

teachers highlight the positive things and give recommendations and suggestions about the things that were not done well so that the teacher can try to change these in the future lessons.
As we mentioned before, Lesson Analysis Checklist can be used to decide which elements should be the main points for discussion and to agree on the targets of the next PTL/LS. Conference notes are to be used for recording the discussion.

The teacher should be thanked and should be appreciated for the learning experience that was provided for everyone and how much the observing teachers have learnt from each other. The teacher should accept what is given as genuine and helpful. The feedback should be taken seriously and the teacher should be given some chance to acknowledge the feedback.

Below is the possible agenda for the Post – PTL/LS Conference.

- Select a facilitator to lead an honest and productive discussion.
- Minimized over personalized feelings.
- The teacher who taught the lesson comments on the lesson
    - What worked and why
    - What did not work and why
    - What should be changed, etc
    - What are the challenges
- Comments by others (Observing teachers)
- Thank the presenting teacher
- Comment on the positive aspects based on concrete evidence
- The teacher reflects on the comments and responds
- Write the report of the activity.

3.1 Minutes of Post – PTL/LS Conference or Lesson Analysis Checklist (Fig 3)

Minutes of a Post – PTL/LS Conference

Date: 14 May 2008 Place: Filini School
Time: 2:00 pm Facilitator: Miss TA

Participating teachers: Mr TH, Mr RO, Miss EW, Miss RP, Mrs MQ and Mrs KS
Topic selected for PTL: Converting Fraction to Percentage and vice versa
Comments by the facilitator:
Comments by the teacher:
What worked well and why:
What did not work and why:

- Introduction was not proper.
- Pressed by the time limit.

What should be changed:
What are the challenges:

Comments by the observing teachers:
What worked well and why:

- The presenter corrected learners' wrong answers in a good way so that the learners could discover what was wrong themselves.
- The presenter was sensitive enough not to demoralize learners who gave irrelevant answers.

- Naming of groups was efficient and effective.

What did not work and why:

- The content was too much for one lesson.
  Introduction of the term "ratio" was confusing because the question presupposes that learners would know the term already.
- Learners needed more attention by the educator.

What should be changed:

- Presenter could have explained the aim and meaning of the classroom observation activity to learners in order to reduce the level of surprise and confusion on their side.

What are the challenges:

- During the presentation, observers should retain themselves from helping the presenter because in reality, teacher must handle everything happening in the classroom alone.

Setting the short term targets:

Comments of the teacher on the feedback:
The presenter generally agrees with the above comments, especially the comment that the amount of contents presented was too much.

Next PTL (PTL2) Date: 7 August 2008 Venue: Filini School Time: 2:00 pm
Responsibility for organizing PTL2: Mrs KS

4. Challenges

There could be several challenges that the teachers may face while organizing school based INSET. For example:

- What happens when the content knowledge of the teachers in the group is low?
- When all the teachers in the group are new to the profession?
- When all the teachers in the group hold traditional views about teaching and learning?
- What happens when teachers are overloaded with teaching periods and extra school work?

To overcome these challenges the expertise of an outside advisor can be sought to provide professional support.

In schools where not one teacher has more expertise in a particular subject in this case mathematics or where schools are small and very few teachers are posted there it may not be practical to convene these sessions at each school, in such cases the teachers should join the sessions organized at a neighboring schools (Cluster).

5. Resources and support for PTL/LS

What are the resources required for PTL/LS? How much does it cost to implement PTL/LS? What kind of support can the teachers encaged in PTL/LS draw on from within and outside their school?

In trying to answer these questions, it is important to recognize that the most crucial resource required for PTL/LS is the knowledge, skills and experiences to be brought forward by the teachers themselves for sharing with their fellow teachers. No two teachers have the same endowment of knowledge, skills and experiences. Some mathematics teachers maybe strong in operation while others in Geometry or measurement. Some teachers may be more experienced in hands on approach to teaching, while some may have unique experiences and backgrounds that may enrich planning of particular lessons. So bring in a group of teachers, and they are bound to have much to share with one another on the basis of their complementarities.

Needless to say, mere existence of complementary resources does not guarantee PTL/LS to occur. More often, this complementarity is not recognized and PTL/LS does not take place at all. In this sense, the most critical resource requirement for PTL/LS is the capacity or initiative of certain teachers to organize PTL/LS. Taking of such an initiative obviously calls for quality leadership from a few innovative teachers. In the school environment, Heads of departments or Subject Heads may be expected to play such a leadership role. However, because of the nature and requirement for the organization of an effective PTL/LS the leadership role must come from not a single teacher but several like minded teachers.

5.1 The role of Heads of Academic Committees & Subject Heads of Departments (HODs) for organizing PTL in schools (Fig 4)

Role of the Training participants and/or HODs
To participate in National, Provincial, Zonal and Cluster – level workshops on behalf of the school in order to learn about new mathematics teaching methods, which can serve as the basis for PTL/LS at the school;
To report back to school and establish a plan for PTL/LS Activities;
To identify and address problems in their department; To develop their colleagues in schools; To establish and develop cooperative team teaching in their schools; To encourage working together in organize cluster meetings; To share their expertise and resources within the school; To share their classroom experiences;

To identify teachers problems in class; To provide quality assurance;
To implement the PTL/LS Plan: • To prepare and convene PTL/LS sessions, including organizing of the Pre – PTL/LS conference, joint elaboration of the lesson plan, designating of the lesson presenter, logistical arrangements, etc • To ensure smooth holding of the PTL/LS lesson presentation and Post – PTL/LS Conference. • To prepare a brief summary of the PTL/LS session. • To encourage classroom application of the results of PTL/LS.
To prepare a quarterly report on PTL/LS activities conducted as school based INSET.

When a group of teachers planning a PTL/LS session finds any deficiency of knowledge, skills and experiences or any needs for learning materials or use of particular equipment
for conducting PTL/LS, whom can they turn to? Where can they look to secure additional inputs?

The school Head should be the first to be contacted and consulted. Situations may vary from school to school. However, officers in the department of Education such as the Provincial Education Officers, the inspectors, Officers of the Curriculum Unit, Zone Curriculum Advisors, Secondary school subject advisors, tutor of Vanuatu Institute of Teacher Education, JOCV Volunteers etc.. are available to provide support or at least assist in securing such support. They may be asked to assist in the entire PTL/LS process with their specialized expertise, be it subject content enrichment or for advising on methods of organizing PTL/LS or may be asked to support in terms of materials for use with PTL/LS session. They should also provide departmental sources of support where assistance could be sought.

6. Conclusion

Peer Teacher Learning should help to promote school based INSET and to enhance professional development of teachers for the successful implementation the Vanuatu curriculum especially in mathematics. In the future when the mathematics curriculum is revised PTL/LS would be a very appropriate approach to use to implement it.

PTL should be an ongoing activity as it has a better chance to make a difference in teacher effectiveness in the classroom. PTL/LS is also in line with main objective of this training to develop a system of school based INSET, to improve the quality of teaching and to enhance learners' skills in Mathematics. PTL/LS is also intended to fine tune the potential of teachers participating in this training.

At times much team work in “Peer Teacher Learning” is necessary to achieve better understanding and to improve the classroom practice of all teachers.

School based INSET should be organized on a continuous basis with increasing elements of curriculum development. In other words, school - based INSET should focus on the developmental characteristics of teachers. This places much emphasis on “team work.” It is hoped that the development of school based INSET for mathematics teachers on a regular basis will improve the capacity and experience of teachers and may evolve into a sustained practice in our schools.

Appendix 11
PTL/LS Conference Minutes form

Pre – PTL/LS Conference Minutes form

Date: .... Place: ....
Time: .... Facilitator: ....

Participating teachers: ....
Topic selected for PTL\LS: ....
Design of the lesson unit: ....
Prior knowledge of the students: ....
....
....

Content to be covered:
....
Intended outcomes:
....
Resources:
....
Lesson activities:
....

Revising the lesson unit:
....

Distribution of the observation tools, and explanation of how to use them.
What to look for:
....

Discussion on the main focus of the observation:

Invitation: ....
Outside assistance:
....
Date for observation:
....
Date for Post – PTL/LS:
....
Facilitator for Post – PTL/LS:
....
Report writing for PTL/LS:
....

Next PTL/LS (PTL/LS 2): Date: .... Venue: .... Time: ....

Responsibility for organizing PTL/LS 2: ....

Appendix 12
LESSON OBSERVATION FORM

School : ……………… Grade : …… Teacher: ……………………… Date : ………

Time Minutes	Describe what the teacher is doing.	Describe what the learners are doing

Post – PTL Conference Minute form

Date:		Place: ....
Time:		Facilitator: ....

Participating teachers: ....
Topic selected for PTL: ....

Comments by the facilitator: ....

Comments by the teacher: ....

What worked well and why: ....

What did not work and why: ....

What should be changed: ....

What are the challenges: ....

Comments by the observing teachers: ....

What worked well and why: ....

What did not work and why: ....

What should be changed: ....

What are the challenges: ....

Setting the short – term targets: ....

Comments of the teacher on the feedback: ....

Next PTL (PTL2)   Date: ....   Venue: ....   Time: ....

Responsibility for organizing PTL2:...

Appendix 13
The development of Geometrical thinking.

Not all people think about geometric ideas in the same manner. Certainly, we are not all alike, but we are all capable of growing and developing in our ability to think and reason in geometric contexts. The research of two Dutch educators, Pierre van Hide and Dina van Hiele-Geldof has provided insight into the differences in geometric thinking and how the differences come to be.
The van Hieles' work began in 1959 and immediately attracted a lot of attention in the Soviet Union but for nearly two decades got little notice in this country (Hoffer, 1983; Hoffer & Hoffer, 1992). But today, the van Hide theory has become the most influential factor in the geometry curriculum.

The van Hiele Levels of Geometric Thought

The most prominent feature of the model is a five-level hierarchy of ways of understanding spatial ideas. Each of the five levels describes the thinking processes used in geometric contexts. The levels describe how we think and what types of geometric ideas we think about, rather than how much knowledge we have. A significant difference from one level to the next is the objects of thought—what we are able to think about geometrically

Level 0: Visualization
The objects of thought at level 0 are shapes and what they "look like."
Students recognize and name figures based on the global, visual characteristics of the figure—a gestaltlike approach to shape. Students operating at this level are able to make measurements and even talk about properties of shapes. but these properties are not abstracted from the shapes at hand. It is the appearance of the shape that defines it for the student. A square is a square "because it. looks like a square." Because appearance is dominant at this level, appearances can overpower properties of a shape. For example, a square that has been rotated so that all sides are at a 45-degree angle to the vertical may now be a diamond and no longer a square. Students at this level wl1 sort and classify shapes based on their appearances—"l put these together because they are all pointy" (or "fat," or "look like a house," or are "dented in sort of," and so on). With a focus on the appearances of shapes, students are able to see how shapes are alike and different. As a result, students at this level can create and begin to understand classifications of shapes.
The products of thought at level 0 are classes or groupings of shapes that seem to be "alike."

Level 1: Analysis
The objects of thought at level I are classes of shapes rather than individual shapes.
Students at the analysis level are able to consider all shapes within a class rather than a single shape. Instead of talking about this rectangle, it is possible to talk about all rectangles. By focusing on a class of shapes, students are able to think about what makes a rectangle a rectangle (four sides, opposite sides parallel, opposite sides same length, four right angles, congruent diagonals, etc.). The irrelevant features (e.g., size or orientation) fade into the background. At this level, students begin to appreciate that a collection of shapes goes together because of properties. Ideas about an individual shape can now be generalized to all shapes that fit that class. If a shape belongs to a particular class such as cubes, it has the corresponding properties of that class. "All cubes have six congruent faces, and each of those faces is a square." These properties were only implicit at level 0. Students operating at level 1 may be able to list all the properties of squares, rectangles, and parallelograms but not see that these are subclasses of one another that all squares are rectangles and all rectangles are parallelograms.
In defining a shape, level 1 thinkers are likely to list as many properties of a shape as they know.
The products of thought at level I are the properties of shapes.

I.evel 2: Informal Deduction

The objects of thought at level 2 are the properties of shapes.
As students begin to be able to think about properties of geometric objects without the constraints of a particular object, they are able to develop relationships between and among these properties. “If all four angles are right angles, the shape must be a rectangle. If it is a square, all angles are right angles. If it is a square, it must be a rectangle.” With greater ability to engage in “if-then” reasoning, shapes can be classified using only minimum characteristics. For example, four congruent sides and at least one right aI1e-can he sufficient to define a square. Rectangles are parallelograms with a right angle. Observations go beyond properties themselves and begin to focus on logical arguments about the properties. Students at level 2 will be able to follow and appreciate an informal deductive argument about shapes and their properties. “Proofs’ may be more intuitive than rigorously deductive. However, there is an appreciation that a logical argument is compelling. An appreciation of the axiomatic structure of a formal deductive system, however, remains under the surface.
The products of thought at level 2 are relationships among properties of geometric objects.

I.evel 3: Deduction
The objects of thought at level 3 are relationships among properties of geometric objects.
At level 3, students are able to examine more than just the properties of shapes. Their earlier thinking has produced conjectures concerning relationships among properties. Are these conjectures correct? Are they “true”? As this analysis of the informal arguments takes place, the structure of a system complete with axioms, definitions, theorems, corollaries, and postulates begins to develop and can be appreciated as the necessary means of establishing geometric truth. At this level, students begin to appreciate the need for a system of logic that rests on a minimum set of assumptions and from which other truths can bederived. The student at this level is able to work with abstract statements about geometric properties and make conclusions based more on logic than intuition. This is the level of the traditional high school geometry course. A student operating at level 3 can clearly observe that the diagonals of a rectangle bisect each other, just as a student at a lower level of thought can. However, at level 3, there is an appreciation of the
need to prove this from a series of deductive arguments. The level 2 thinker, by contrast, follows the argument but fails to appreciate the need.
The products of thought at level 3 are deductive axiomatic systems for geometry.

Level 4: Rigor
The objects of thought at level 4 are deductive axiomatic systems for geometry.

At the highest level of the van Hiele hierarchy, the objects of attention are axiomatic systems themselves, not just the deductions within a system. There is an appreciation of the distinctions and relationships between different axiomatic systems. For example, spherical geometry is based on lines drawn on a sphere rather than in a plane or ordinary space. This geometry has its own set of axioms and theorems. This is generally the level of a college mathematics major who is studying geometry as a branch of mathematical science.
The products of thought at level 4 are comparisons and contrasts among different axiomatic systems of geometry.

Characteristics of the van Hiele Levels

You no doubt noticed that the products of thought at each level are the same as the objects of thought at the next, This object-product relationship between levels of the van Hiele theory is illustrated in Figure 20.2. The objects (ideas) must be created at one level so that relationships among these objects can become the focus of the next level. In addition to this key concept of the theory, four related characteristics of the levels of thought merit special attention.

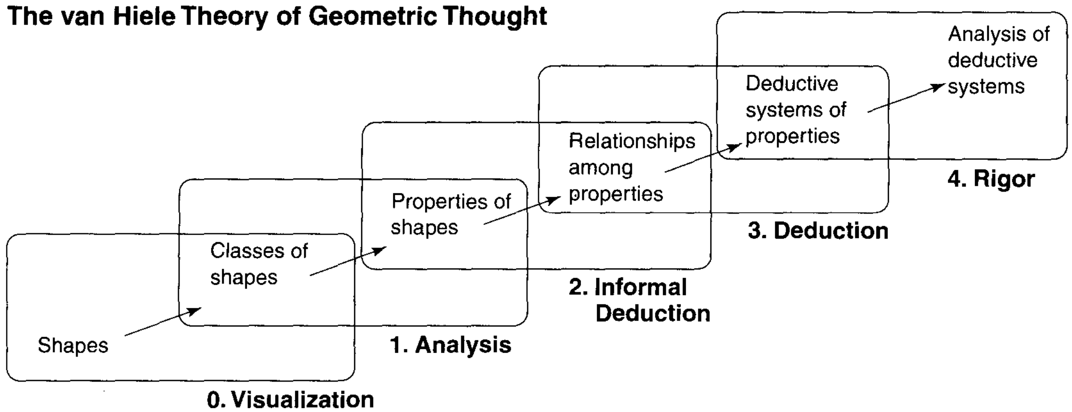
1. The levels are sequential. To arrive at any level above level
0, students must move through all prior levels. To move through a level means that one has experienced geometric thinking appropriate for that level and has created in one's own mind the types of objects or relationships that are the focus of thought at the next level.

2. The levels are not age-dependent in the sense of the developmental stages of Piaget. A third grader or a high school student could be at level 0. Indeed, some students and adults remain forever at level 0, and a significant number of adults never reach level 2. But age is certainly related to the amount and types of geometric experiences that we have. Therefore, it is reasonable for all children in the K—2 range to be at level 0, as well as the majority of children in grades 3 and 4.

3. Geometric experience is the greatest single factor influencing advancement through the levels. Activities that permit children to explore, talk about, and interact with content at the next level, while increasing their experiences at their current level, have the best chance of advancing the level of thought for those children.

4. When instruction or language is at a level higher than that of the student, there will be a lack of communication. Students required to wrestle with objects of thought that have not been constructed at the earlier level maybe forced into rote learning and achieve only temporary and superficial success. A student can, for example, memorize that all squares are rectangles without having constructed that relationship. A student may memorize a geometric proof but fail to create the steps or understand the rationale involved (Fuys, Geddes, & Tischler, 1988; Geddes & Fortunato. 1993).

The van Hiele Theory of Geometric Thought



The van Hide theory does not tell us what content to teach, but it does provide the thoughtful teacher with a framework in which to conduct geometric activities. The district or state in which you teach will prescribe the specific content of your geometry curriculum. You should be clear about both content goals and the goals implicit in the van Hide theory

Appendix 14
TRAINING EVALUATION FORM

Please evaluate the workshop by filling the spaces below.

What I liked about the workshop?

What did I not like?

The following need improvement:

Other suggestions:

References

Booker,G., Bond,D., Buggs,J., and Davey, G., (1997) Teaching Primary Mathematics. Addison Wesley Longman Aust Pty Ltd:Melb

English Translations of Japanese Course of Study in Mathematics

http://www.criced.tsukuba.ac.jp/jocv-2006/material/CRICED_TeachingGuide_H10.pdf

http://www.criced.tsukuba.ac.jp/jocv-2006/material/CRICDE_teachingGuide.pdf

Masami I, Max S. Yutaka O, Takeshi M.(2207) Japanese Lesson Study in Mathematics: *Its impact, Diversity and potential for educational improvement, Pte Limited*

http://www.gakuto.co.jp/20050131e/index.html

Masami I.(2006). Developing Mathematical thinking in the classroom.

Masami I.(1996).Where do mathematics problems come from at elementary school classrooms? Dialectic discussion beyond contradictions in the classroom on the Problem Solving Approach.

[http://www.criced.tsukuba.ac.jp/math/apec/apec2007/progress_report/specialists_session /Masami_Isoda.pdf](http://www.criced.tsukuba.ac.jp/math/apec/apec2007/progress_report/specialists_session/Masami_Isoda.pdf)

Robert J S (1996) Teaching mathematics to children. Addition Wesley Education Publishers Inc

Stigler, J. W., & Hiebert, J. (1999). The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom. New York, NY: The Free Press.

Van de walle. J. A (2004) 5th Edition. Elementary and middle school mathematics: Teaching Developmentally . Pearson Education. Inc

Videos of Japanese Mathematics Classes;

http://www.criced.tsukuba.ac.jp/math/video/