Hermeneutics of Historical Mathematics Textbooks

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Euclid's elements: the first printed Greek edition 1533


ΕΥΚΛΕΙΔΟΥ
ΣΤΟΙΧΕΙΩΝ ΒΙΒΛ▸ ΙΕ▸
ΕΚ ΤΩΝ ΘΕΩΝΟΣ ΣΥΝ,
ΟΥΣΙΩΝ.

Εἰς τὰ αὐτὰ τὸ πρῶτον, ἐξηγημάτων Πρόκλου βιβλία.

Adiecta præfatiuncula in qua de diſciplinis Mathematicis nonnihil.

BASILEAE APVD IOAN. HERVAGIVM ANNO M. D. XXXIII. MENSE SEPTEMBRI.

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The Wrong Text of Euclid: On Heiberg's Text and its Alternatives*

by

WILBUR R. KNORR**

> The stone which the builders refused is become the headstone of the corner. *Psalms* (AV) 118:22

Introduction

Over a century ago two young academics engaged in a brief debate. One, a Gymnasiallehrer at Altona in Germany, M. Klamroth, was a specialist in medieval Arabic mathematics. The other, J. L. Heiberg, only five years beyond his doctoral dissertation at Copenhagen, had already completed a new critical edition of the Greek text of Archimedes and was in the process of issuing a new critical

Two directions of transmission of Euclid's Elements: north the Mediterranean (Greek versions) and south: Arabic versions - from Alexandria to Western Europe

Remarques sur l'Histoire du Texte des Éléments *d'Euclide*

S. ROMMEVAUX, A. DJEBBAR et B. VITRAC*

Presenté par A. DJEBBAR *et* B. VITRAC

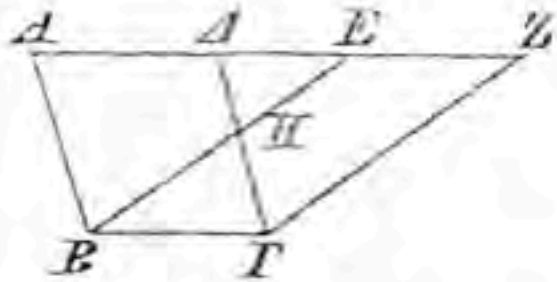
Introduction

Éditer un texte antique suppose d'en trouver les manuscrits conservés, de recenser leurs variantes, d'en établir une généalogie pour détecter des filiations entre copies en essayant de se rapprocher autant que faire se peut de l'autographe original. En dehors des manuscrits portant une version du texte qu'il veut éditer ou une partie de ce texte, voire des papyri anciens quand ils existent – on appelle cet ensemble **tradition directe** – le philologue a recours à d'autres textes: les citations par d'autres auteurs, les commentaires anciens quand ils existent, les traductions anciennes... qui constituent ce qu'on appelle habituellement la **tradition indirecte**. Parfois cette dernière permet d'obtenir des informations relatives à une période plus ancienne que la tradition directe, entièrement déterminée par une réédition de l'Antiquité tardive. Mais que faut-il penser quand les traductions, par exemple, suggèrent un état du texte sensiblement différent de celui (ceux) que nous fait connaître la tradition directe des manuscrits, au demeurant soumis aux aléas de la transmission par copies successives?

Les *Éléments* d'Euclide sont incontestablement le texte mathématique grec ancien dont la tradition indirecte est la plus riche et ceci n'est pas sans rapport avec le succès du traité durant une longue période, qui va de l'Antiquité tardive à la fin du XVI^{e} siècle, succès qui s'exprime dans les différentes langues des cultures anciennes et médiévales:

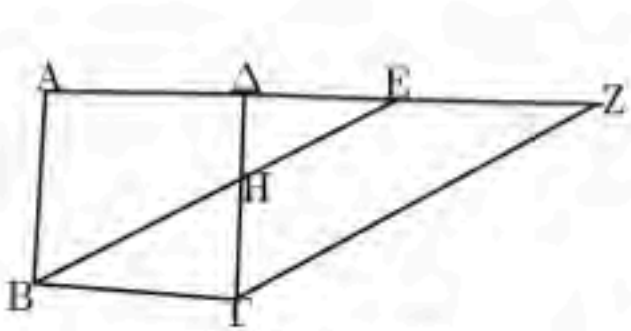
Ken Saito: research on the figures in Euclid versions

- - differences between autograph versions (Middle Ages) and later printed versions, in particular since the 19th century
- Autographs: missing attention for mathematical properties – p.e.: a rectangle instead of a parallelogram: “hyper-specification”

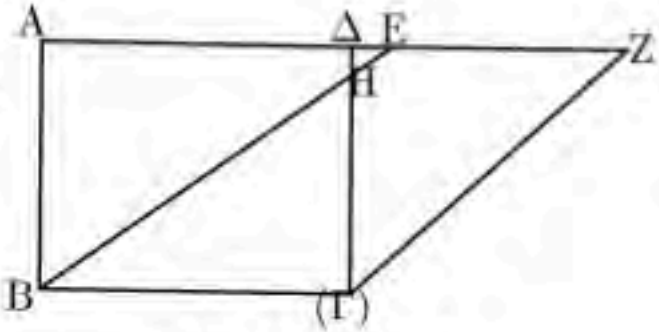


I.35 Heiberg

Fig. 1

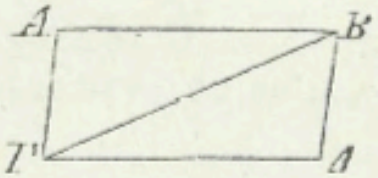


P: prop. 01-35



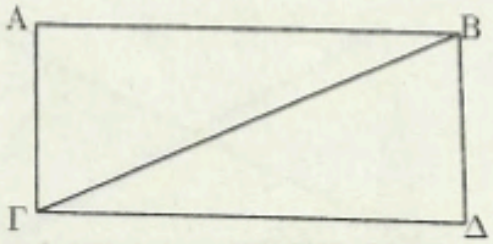
B: prop. 01-35

Proposition I.34

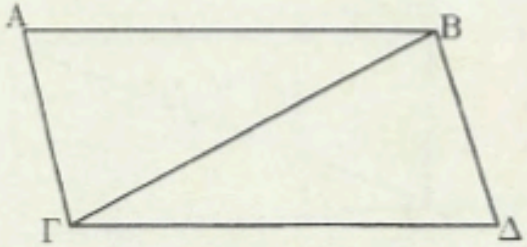


Heiberg

GB GR: The diagrams are rotated, not horizontally reflected.



Codex P



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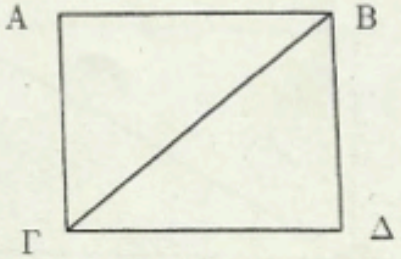

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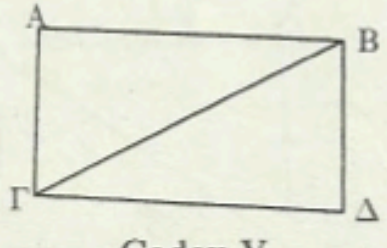
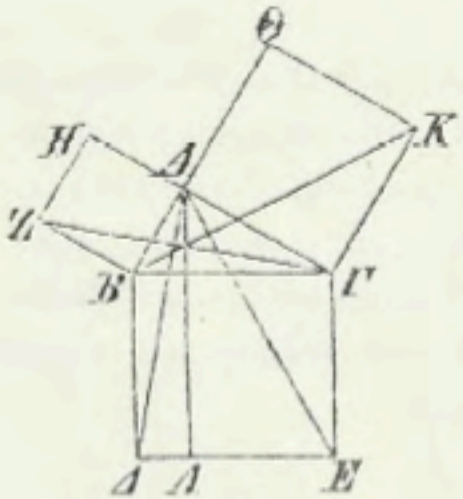


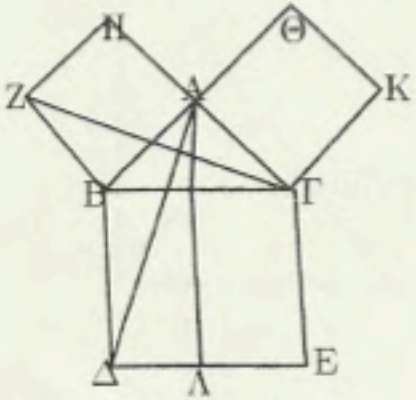
Codex V

Proposition I.47



Heiberg

V: Lines AE and BK are drawn freehand. Numbers are written by more recent hands. **PV:** For three sides 5, 5, 7;4,15,50 (**V:** 7;4,15), and for three squares 25, 25, 50, all in Arabic numerals. **b:** 5, 5 for sides of upper squares and 25 for their areas in Greek numerals. **B:** 3, 4, 5 for sides and 9, 16, 25 for squares in Greek numerals; the same set of numbers appear in diagrams in the margin of **P**(Greek), **V**(one diagram in Greek, another in Arabic). These mss. contain other writings that I have not been able to decipher.



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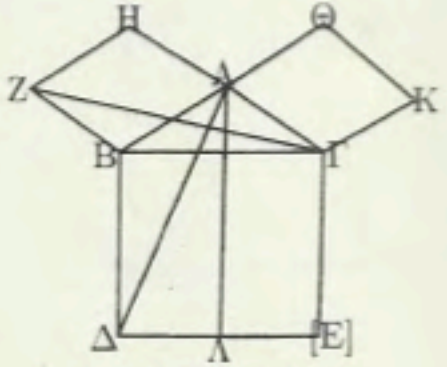

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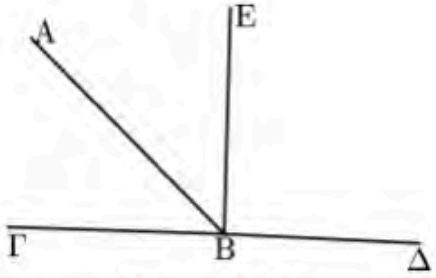
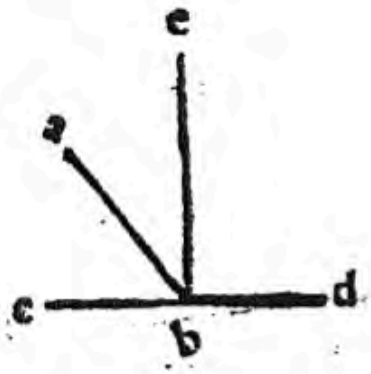
Few attention to details

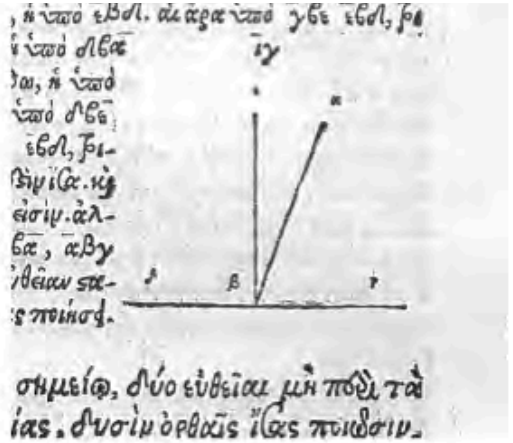


P: prop. 01-13

Codex P

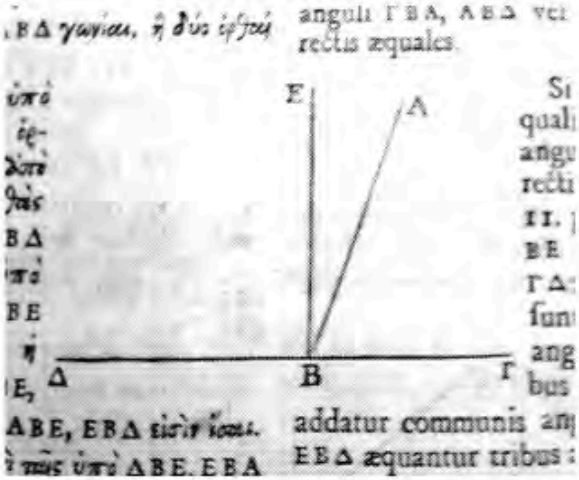


Zamberti

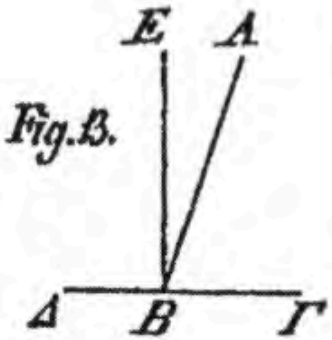


σημείῳ, δύο εὐθεῖαι μὴ πλεῖ τὰ
ίας. δυσὶν ὀρθαῖς ἴσας ποιοῦσι.

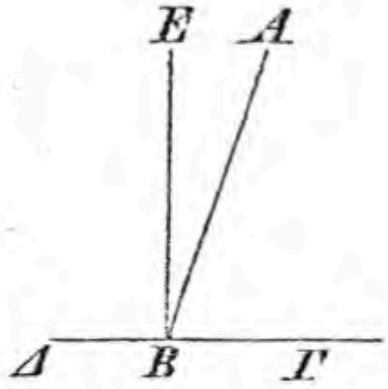
Grynaeus



Gregory



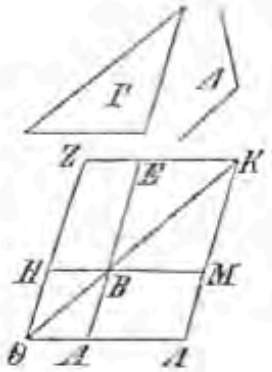
August



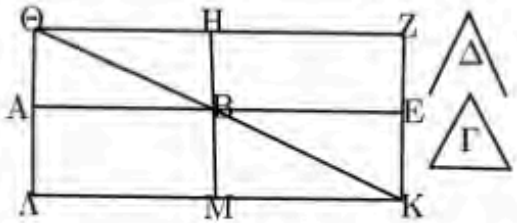
Heiberg

Even incorrect figures: I.44 – area *G* (Γ) has to be equal to parallelogram *BEZH*

Proposition I.44

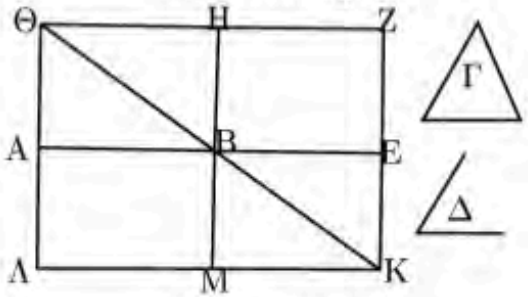


Heiberg



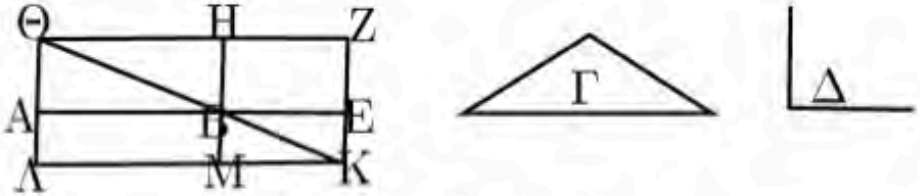
P: prop. 01-44

Codex P



B: prop. 01-44

Codex B



V: prop. 01-44

Codex V

Fig. 11

Assessment of the role of figures by Saito

- They contain too much information that are not guaranteed by the premise of the proposal, and one is concerned that readers (and students if one is a teacher) can be mistaken. Sometimes the figures are even incorrect, conflicting with the information given in the text. They are not metrically correct, and often so they are so schematic that they seem to express something else, especially in solid geometry.
- Figures with all these special characteristics suggest that their role was much more limited than what we are used to impose on them. They are what we call today topological or schematic representations of spatial relationships, and the reader must be able to read the figures according to the specifications given in the text. Modern editors, except a few among the first in the sixteenth century, have shown a constant predilection for general and correct figures, which culminates in the didactic edition of August, copied almost entirely by Heiberg.

Hermeneutical issue implied:

- How general are figures?
- Saito: “The fact that many editors [of manuscripts] have liberally and without difficulty altered the figures is also evidence of the limited role of the figures. If the text is correct, there is a wide range of possibilities for the figures”.
- Figures represent always concrete situations – that is they appear to be specific
- But they have to represent a general case

- This issue implies a methodological problem for teaching: teaching so that the student can succeed in abstracting from the concrete positions

Additional, more radical question

- Did the original text of Euclid's Elements contain figures?
- Original form of presentation of a text: oral
- Thus: maybe constructed ad hoc, during the lecture?
- In fact. Usually, the text of propositions is formulated in a manner to indicate how the figure has to be constructed – and this would explain the differences in the old autograph versions

Another hermeneutical issue: assessing pedagogical approaches in textbooks

- Controversial example: Alexis-Claude Clairaut (1713-1765) – Élémens de géométrie, 1741

- *I intended to go back to what might have given rise to geometry; and I attempted to develop its principles by a method natural enough so that one might assume it to be the same as that of geometry's first inventors, attempting only to avoid any false steps that they might have had to take...*

TABLE DES MATIERES.

PREMIERE PARTIE.

Des moyens qu'il étoit le plus naturel d'employer pour parvenir à la mesure des Terrains.

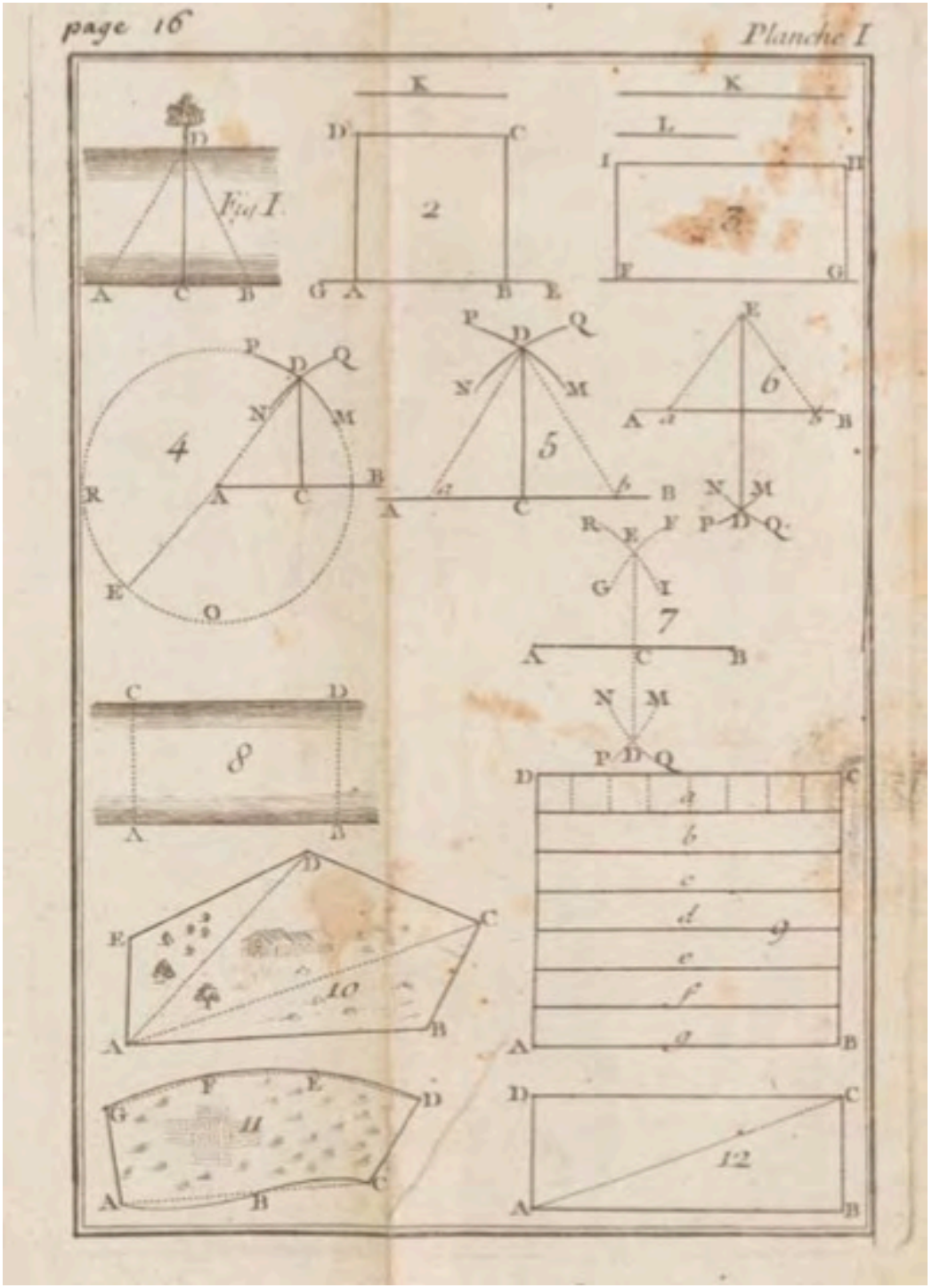


Fig I.
2
3
4
5
6
7
8
10
11
12

CLAIRAUT

ELEMENTOS

DE

GEOMETRIA

TRASLADADOS A PORTUGUEZ

POR

JOSÉ FELICIANO

S. PAULO

EMPREZA BIBLIÓPOLA—EDITORA

1892

(4ª da Republica Centésimo quarto da Revolução Franceza)

a three-dimensional approach to achieve pertinent results in textbook analysis:

- the first dimension consists in analysing the changes within the various editions of one textbook chosen as the starting-point, say an algebra textbook, or an arithmetic one;
- the next dimension consists in finding corresponding changes in other textbooks belonging to the same oeuvre, by studying those parts dealing with related conceptual fields, say geometrical algebra, trigonometry, etc.
- the third dimension relates the changes in the textbooks to changes in the context: changes in the syllabus, ministerial decrees, didactical debates, evolution of mathematics, developments in epistemology, etc.

IV. Lacroix as textbook entrepreneur – the sequence of his algebra textbook

- First "curriculum for the French secondary schools (*lycées*), in 1803: a list of textbooks:
- Lacroix Traité élémentaire d'arithmétique
- Lacroix Éléments de Géométrie
- Lacroix Traité élémentaire de trigonométrie et de l'application de l'Algèbre à la géométrie
- Lacroix Compléments des éléments d'algèbre
- Francoeur Traité élémentaire de mécanique

Social context of this decision

- Lacroix acting in the Ministry of the Interior (competent for instruction, too)
- Lacroix member of the commission, which had to elaborate the curriculum – together with Laplace and Monge

The algebra edition seqeunce

- **Version no. 0**: re-edition (anonymous) **1797** of Clairaut's algebra
  - Social context: demands of public instruction not yet conscious
  - **Edition no. 1: 1799**. Based in Bézout, first critic of Clairaut

  - Social context: Bézout most used by the math teachers and propagated by Lagrange
  - **Re-edition, no. 2: 1800**: identical, but more critic of Clairaut in the preface – not suited for public instruction
  - **Edition no. 3: 1803**. Complete revision, no more based on Bézout. Proper text – adopting Carnot's conceptions

  - Social and epistemological context: impact of Carnot's rejection of negative quantities

ÉLÉMENS
D'ALGÈBRE,

A L'USAGE
DE L'ÉCOLE CENTRALE
DES QUATRE-NATIONS,

PAR S. F. LACROIX.

CINQUIÈME ÉDITION,
revue et corrigée.

A PARIS,
Chez COURCIER, Imprimeur-Libraire pour les Mathématiques, quai des Augustins, n° 71.
AN XIII = 1804.

1797: adhering to Clairaut and d'Alembert

- Why had Lacroix opted for not writing a proper textbook and instead decided to re-edit Clairaut's book? It is obviously an impact of the conceptions divulgated by d'Alembert who had emphasized the method of the inventors as the best method for composing textbooks. In fact, Lacroix – in his anonymous preface outed himself as an adherent of the method of the inventors:
- "The Elements of Algebra Clairaut, in which the readers participate, somehow, in the invention of the analysis, contain many and varied applications; they require, moreover, very few acquaintances in arithmetic, and this advantage is valuable to many readers since the study of this part treated somewhat too extensive in textbooks, has them scared off".

Reservations of Lacroix regarding Clairaut's method

- Clairaut's book being far from containing all what is useful in algebra – it being only a first preliminary which needed to be complemented. In fact, he added a series of additions, but did not credit himself with their authorship, but famous mathematicians like Euler, Lagrange and Laplace. Lacroix emphasized that there were two different types of additions:
- - simple notes, which resumed several particular results in a more general form – thus already implicitly criticisinf the one-sidedly practical orientation to solve problems, missing to generalize the results;
- - and secondly, theoretical developments, which were altogether missing in Clairaut's book.

1799: other copies

- It proves to constitute a break with d'Alembert's notion oft he method of the inventors. Lacroix had eliminated Clairaut's text entirely and substituted it by another, still not of his own authorship: however, of Bézout! But also complemented: again by texts copied from Euler, Lagrange and Laplace – this time, however, also "some new pieces" – and these by him himself. Lacroix justified his removal of Clairaut by it being very long, and praised, in contrast, the "speed" of Bézout. Lacroix admitted the inaccuracy of Bézout's demonstrations, and claimed to have corrected such failures. Lacroix also apologized for not writing an entire textbook alone, saying he had not had time to do it.

1800: re-edition of 1799, but with more critique on Clairaut

- still without explicit indication of him as author, Lacroix now openly criticized the methodology to follow the path of the inventors, which had been dominant so far, giving him only a limited value of motivation for the start of a textbook. Although the text was essentially identical with the previous edition, Lacroix devoted most of his own preface to explain the deficiencies in the way of the inventors in general and of Clairaut's textbook in particular. Lacroix said to have convinced himself of the tedious prolixity of this method and that its value is restricted to the first basics.

1803 4th (3rd) edition: radical change

- for the first time contained a text apparently written exclusively by Lacroix himself. From this new edition on, Lacroix had also eliminated the copies from Bézout's text, replacing it with his own text. There remained no trace of the previously so revered method of the inventors. After a several years' experience adminstering a public school system, Lacroix had understood that such a "genetic" method was not applicable to public education; in order to generalize the knowledge in a better way, Lacroix now preferred the direct development of theories and their applications, an approach he called "dogmatic".

Impact of Carnot: reorganised due to refusal of negative numbers

- the existence of negative amounts was no longer admitted, there was no reflection on the related operations. Negative solutions were disqualified as “absurdities”
- two epistemological reasons for the rupture:
- - the operation of subtraction is restricted to the case of a positive "remainder". (Lacroix, 1803, p. 92)
- - the underlying conception of quantities proves to be that of concrete quantities, not of abstract quantities or numbers; in particular, the quantities considered are *francs* – the French currency, so that in fact negative *francs* do not make sense. For example, a discussion of a linear system of two equations, over two or more pages, with two quantities *x* and *y* apparently abstract ones, such as 60x + 7y = 46 abruptly terminates with a concrete solution in the quantities: “*x* = 5 fr, *y* = 2 fr”

VI. Lacroix's fight with Legendre's geometry textbook

- In February 1799, Legendre learned that Lacroix would publish a geometry textbook. He worried seriously about the fact that his own geometry book, which was hitherto the only modern French text, could be threatened by a dangerous competitor. So Legendre proposed Lacroix a meeting, at which he urged him to give up publishing the planned geometry volume. Lacroix agreed and promised to do so; however, this would not be his last word on the subject. Three days later, Duprat, Lacroix's editor (as well as of many other important texts of Mathematics) was to meet Legendre: Lacroix had not only given up the geometry text but had also withdrawn his arithmetic and algebra books.

Legendre ceded to the pressure

- Duprat complained to Legendre about the huge economic loss he would incur from the withdrawal of three works at once. Legendre replied with a letter to Lacroix, dated February 16, 1799, which reaffirmed the commitment to smoothen the abandonment of Lacroix's own project: Lacroix should continue to use Legendre's geometry textbook in his position as professor at the *Ecole Centrale*, but was free to "complement" the text "orally" and "to develop the material by means of other works" of his own writing:
- "But since this sacrifice costs you too much, and since it is too expensive for the Citizen Duprat, and since between three bidders it is better than only one to be sacrificed instead of two, I willingly agree that it is me who becoms sacrificed. [...] Thus consider the promise you made me three days ago as not having taken place, and I give you the freedom to publish its own geometry. "

Social context: why a need for a “complete” textbook series?

- Lagrange’s activities in the deliberations of a committee in 1799 to define the textbooks provide the key to understanding why Lacroix insisted also publish their own geometry textbook, and because he instigated Duprat to intervene on his behalf with Legendre: the criterion of Lagrange to choose a textbook was whether this was "complete", that is, whether it covered all the traditional sub-disciplines of mathematics. In late 1798 and early 1799, Lacroix seems to have been extraordinarily pressed to publish his own textbooks.

VII. Legendre's fame and critiques and alterations

- Context: French Revolution and priority for modern textbooks:
- Legendre's geometry was conceived and published as a contribution to one of the most remarkable and innovative projects of the young French Republic, to realize a key intention of the French Revolution: eliminate the prejudices, which had kept the population in ignorance, by disseminating modern scientific knowledge in the broadest way. In order to form the basis for a new public education system, the parliament decided as a first step, in 1794, to open a competition for elementary textbooks.

Appraisals of Legendre's work

- His book won "distinction" by the júri in the competition for "elementary" books: it inaugurated a new textbook style and overcame the earlier pre-didactic models.
- Delambre 1810: " Legendre undertook it to revive among us the predilection for rigorous demonstrations".
- Cajori 1890: "Legendre hás been the greatest modern rival of Euclid"
- August Leopold Crelle, German mathematician, journal editor and translator of Legendre: praised the rigor of his geometry

ÉLÉMENTS
DE
GÉOMÉTRIE,
AVEC DES NOTES.

Par ADRIEN-MARIE LEGENDRE.

Si quid novisti rectius istis,
Candidus imperti.

A PARIS,

Chez FIRMIN DIDOT, rue de Thionville, N°. 116,
Libraire pour les Mathématiques et l'Architecture.

L'an IIe de la République.

M. DCC. XCIV.

Legendre striving for improving rigor

- During Legendre's lifetime (1752-1833) 12 editions published of his geometry, always striving to improve its rigor
- In particular: improving the demonstration of the Parallel Postulate, which he – in concordance with the epistemology domination in France – thought to be provable

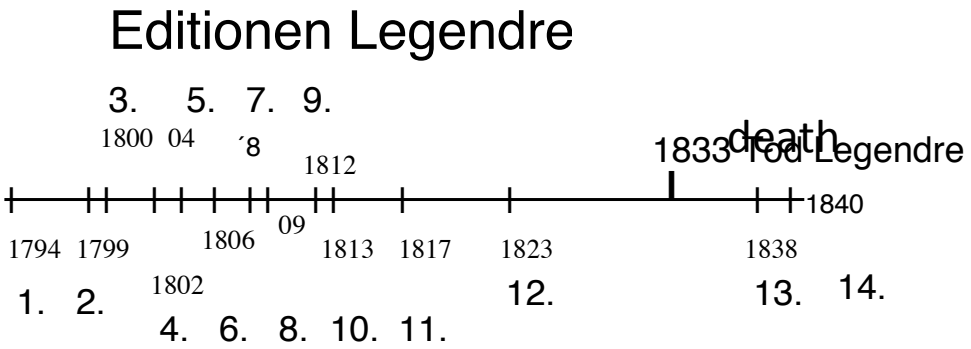
Key point: role of the theory of proportions (to understand later critics)

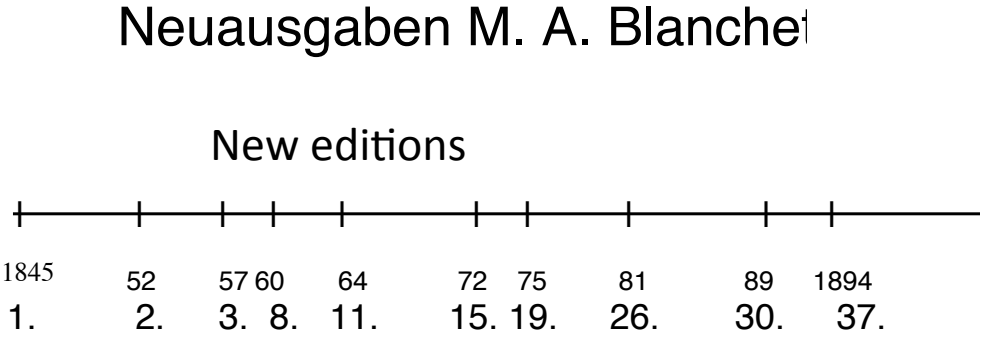
- This means: relation between arithmetic/algebra and geometry: Legendre emphasized the historical character of the theory of proportions: the Greeks have not yet developed algebra and so they made use of proportions; but today one no longer needs to apply this approach. Thus, he did not need to expose in his book – as Euclid had to do - the theory of proportions; instead, he could assume the reader: either being familiar with the old theory of proportions or with the fundamentals of arithmetic and algebra

Made algebra and geometry interacting

- - “For us, who have this instrument [algebra] at hand, we should have no reason not to use it, if it could result in greater ease. I did not hesitate, then, to employ the signs and operations of algebra when I judged it necessary”.
- - “I think, finally, that if the study of geometry should be preceded by some algebra lessons, it is not futile to proceed with studying sciences and to interact them as soon as possible. As one advances in geometry, there is the need to combine a greater number of sets of reasons, and algebra can be of great help to lead to results in a faster and easier way. [...]. Thus I used algebra to combine these proposals with each other and deduce different cases.”

Blanchet: an unknown dared to mutilate Legendre





ÉLÉMENTS
DE GÉOMÉTRIE

PAR A. M. LEGENDRE,

NOUVELLE ÉDITION,

AVEC ADDITIONS ET MODIFICATIONS,

PAR M. A. BLANCHET,

Ancien élève de l'École polytechnique,
directeur des études mathématiques de Sainte-Barbe.

SUIVIE

DE LA QUATORZIÈME ÉDITION,

DONNÉE

PAR A. M. LEGENDRE,

MEMBRE DE L'INSTITUT ET DE LA LÉGION D'HONNEUR,
DE LA SOCIÉTÉ ROYALE DE LONDRES, ETC.

PARIS,

LIBRAIRIE DE FIRMIN DIDOT FRÈRES,

IMPRIMEURS DE L'INSTITUT DE FRANCE,

RUE JACOB, 56.

1845.

The mutilations by Blanchet

- Rarity: the first two (re-)editions of 1845 and 1847 were doubled:
- Blanchet's version and Legendre's last version: 285 + 271 pages! Original skipped from the 3rd edition on. Proper numeration of the Blanchet editions
- Preface: spoke of "imperfections and lacunae" in Legendre

Different conception of geometry, from the beginning

- Legendre started from a general description of what is the geometry: According to Legendre, geometry is a science that has as its subject to measure extensions. Consequently, he introduced the three dimensions of extension: length, width and height in order to characterize the three basic concepts of geometry:
  - Line (length without width); surface (having length and width, but not height): solid or body (having the three dimensions of extension).

ÉLÉMENTS
DE GÉOMÉTRIE.

LIVRE PREMIER.

DÉFINITIONS.

I. Tout corps occupe dans l'espace indéfini un lieu déterminé qu'on appelle *volume*.

II. La *surface* d'un corps est la limite qui le sépare de l'espace environnant.

III. Le lieu où les surfaces de deux corps se rencontrent est appelé *ligne*.

IV. Un *point* est le lieu où deux lignes se coupent.

V. On conçoit les volumes, les surfaces, les lignes, indépendamment des corps auxquels ils appartiennent.

VI. On donne le nom de figures aux volumes aux surfaces et aux lignes.

VII. La *géométrie* a pour objet la mesure de l'étendue des figures, et l'étude de leurs propriétés.

VIII. La *ligne droite* est une ligne indéfinie qui est la plus courte entre deux quelconques de ses points.

On doit regarder comme évident que d'un point à un autre on ne peut mener qu'une ligne droite, et que si deux portions de lignes droites coïncident, ces lignes coïncident dans toute leur étendue.

Limits as basic concept in geometry

- However, Blanchet started from the notion of solid and volume and deduced the notion of surface as the boundary that separates the solid from the surrounding space and the line as the meeting of the surfaces of two solids. The point was the place where two lines intersect.
- The use of the term 'limit' was not by chance. Indeed, already in the preface Blanchet mentioned as one of his decisive changes to have replaced, in the propositions about circles and round solids, the method of proving ad absurdum by demonstrations "by the method of limits", even if this method belonged to higher mathematics!
- Therefore a section on the method limits introduced.
- Mutilated the proofs differentiaiting between commensurable and incommensurable cases.

Broadest international dissemination

- 1794 Original
- 1802 Italy
  - 1807 Spain
  - 1809 Brazil
    - [1810/1812 Greece]
    - 1819 USA
      - 1819 Russia
      - 1822 England
        - 1822 Germany
        - 1826 Sweden
          - 1829 The Netherlands
            - ca. 1830 Switzerland
              - 1836 Ottoman Empire

Enormous success in Italy

PISA
1802
Orig. Leg.

FLORENZ
[G. Cellai]
Orig. Legendre

Gius. François
Blanchet

Adamo Bisse
[Blanchet]

Giulio Tolomei
[Blanchet]

LIVORNO
Gaetano Cellai
Legendre 12./13. A.

LUCCA B. Poli
nach Blanchet

NEAPEL
T. Mandoj
Legendre 12. A.

G. Zocchi

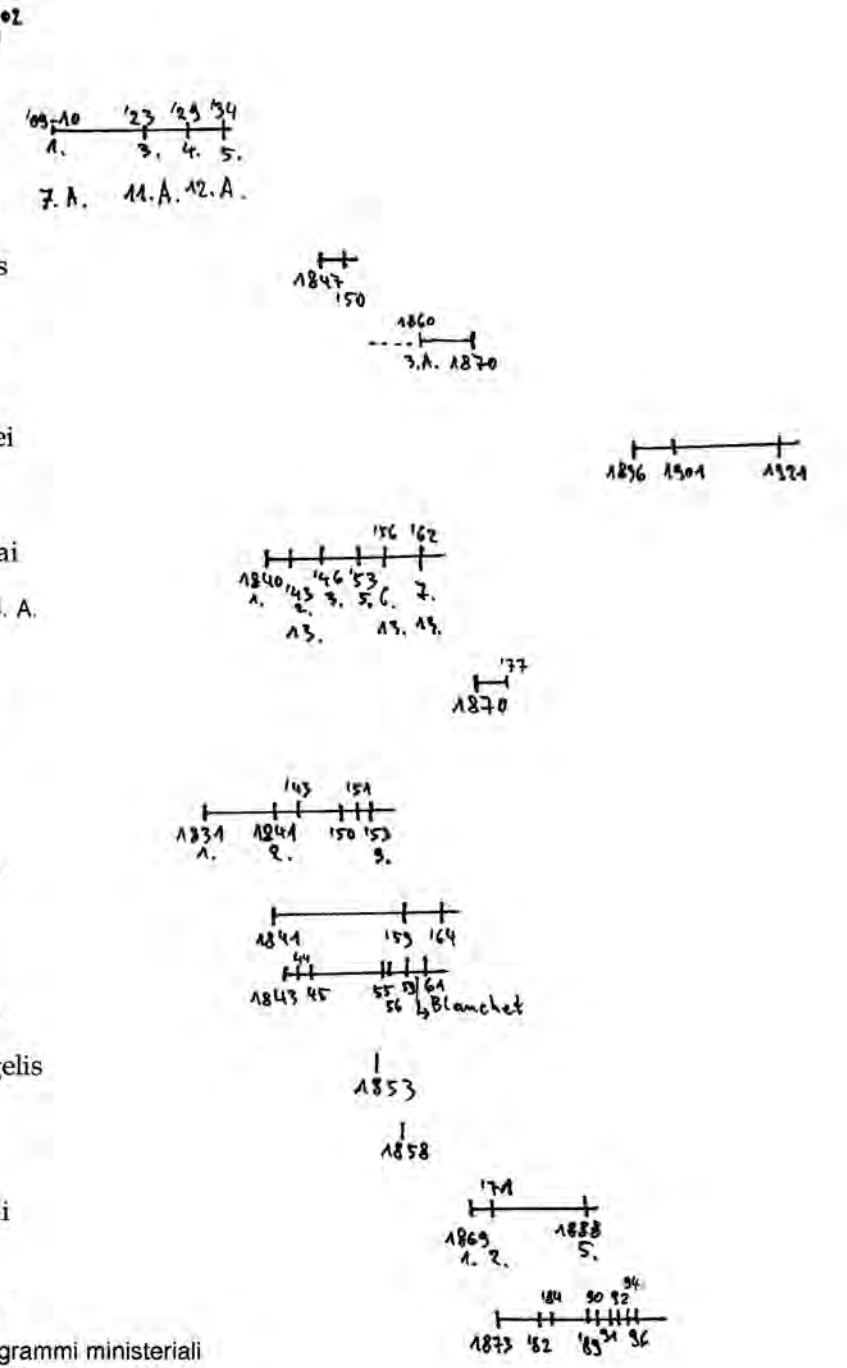
R. Rubini
Legendre 20. A.

Enrico de Ângelis

V. Panunzio
Blanchet

M. Rinonapoli
Blanchet

Gius. Hueber
Blanchet,
conforme ai programmi ministeriali



Refusal in Italy from 1867

- Due to the mathematician Luigi Cremona, president of the curriculum committee for the unified Italy:
- 1. nationalism, against “foreign textbooks”
- 2. to parallelize the teaching of mathematics with that of classical languages in the secondary schools: mathematics being classic too, i.e. like deductive geometry in Greece, thus serving as “mental gymnastics” – while eliminating all applied aspects
- 3. lack of rigor: mixing geometry with algebra

Arguments by Cremona, Betti and Brioschi

- Euclid's geometry being complete and auto-sufficient, not needing support by the science of numbers. Geometry had to be taught in exactly the Greek terms, i.e. with proportions and never using numbers. Therefore Legendre not suited for this kind of methodological objectives.
- "The purity of the Geometry of the Ancients should not be blurred by transforming geometric theorems into algebraic formulas"
- Doubt: were they speaking of the Blanchet-mutilation?

English editions in the USA by Charles Davies (1798-1876): bestseller

- 1828 to 1896: 50 editions
- Based on Legendre's 10th edition, but in the adaptation of the English translation by Carlyle/ Brewster of 1819: already with the introduction of a section with the theory of proportions.
- Davies substituted that section by even a chapter on proportions. And many more changes, "for the use in the USA"

ELEMENTS

OF

GEOMETRY AND TRIGONOMETRY

FROM THE WORKS OF

A. M. LEGENDRE

ADAPTED TO THE COURSE OF MATHEMATICAL INSTRUCTION
IN THE UNITED STATES

BY CHARLES DAVIES, LL.D.
AUTHOR OF A FULL COURSE OF MATHEMATICS

EDITED BY

J. HOWARD VAN AMRINGE, A.M., PH.D.
PROFESSOR OF MATHEMATICS IN COLUMBIA COLLEGE

NEW YORK ·:· CINCINNATI ·:· CHICAGO
AMERICAN BOOK COMPANY

Conceptual changes

- Claimed that his version be more general than that of Legendre!
- - regarding the use of figures in demonstrations

PREFACE.

OF the various treatises on Elementary Geometry which have appeared during the present century, that of **M. Legendre** stands pre-eminent. Its peculiar merits have won for it not only a European reputation, but have also caused it to be selected as the basis of many of the best works on the subject that have been published in this country.

In the original treatise of **Legendre**, the propositions are not enunciated in general terms, but by means of the diagrams employed in their demonstration. This departure from the method of **Euclid** is much to be regretted. The propositions of Geometry are general truths, and ought to be stated in general terms, without reference to particular diagrams. In the following work, each proposition is first enunciated in general terms, and afterward with reference to a particular figure, that figure being taken to represent any one of the class to which it belongs. By this arrangement, the difficulty experienced by beginners in comprehending abstract truths is lessened, without in any manner impairing the generality of the truths evolved.

Role of figures

- See the discussion at the beginning. Is in reality acritique of the earlier US-translation by Farrar: figures there – different from Legendre's original – not in separate tables at the end, but included into the text pages.
- Of course, it is surprising to note that he required for the USA - where the teaching of mathematics was still in its beginnings – to achieve a greater generality and abstraction than in countries already more developed. Davies himself has even used the figures of Legendre. Besides this his critique is not valid: the text of Legendre relies on figures only to visualize the geometric situation, but the demonstrations are not based on the use of figures. Davies's criticism may seem legitimate when one makes a superficial reading of the concurrent version by Farrar. Thanks to a new technique of printing, the figures were incorporated into his text, no more separated at the end. The figures are in fact in front of their propositions. Besides this, in a way that may seem mistaken, Farrar refers directly to neighboring figures - but it is clear that he does not want to assimilate the particular to the general.

Davies dares to criticize Legendre's terminology as not suited

- - and more intrusions
- - definitions "revised"
- - demonstrations "harmonized" and "abbreviated": clearly loss of rigor!

ality of the truths evolved.

The term *solid*, used not only by Legendre, but by many other authors, to denote a limited portion of space, seems calculated to introduce the foreign idea of matter into a science which deals only with the abstract properties and relations of figured space. The term *volume* has been introduced in its place, under the belief that it corresponds more exactly to the idea intended. Many other departures have been made from the original text, the value and utility of which have been made manifest in the practical tests to which the work has been subjected.

In the present edition, numerous changes have been made, both in the Geometry and in the Trigonometry. The definitions have been carefully revised—the demonstrations have been harmonized, and, in many instances, abbreviated—the principal object being to simplify the subject as much as possible, without departing from the general plan. These changes are due to Professor Peck, of the Department of Pure Mathematics

Conclusion

- No text speaks for itself – a hermeneutical analysis is needed, to reveal the intended meaning within its contemporary cultural and social context.
- Schoolbooks might seem to be easy to analyze, but in reality it constitutes a conceptual and methodological challenge to unravel the meaning hidden among a lot of layers.

References

D'Alembert, Jean le Rond (1755), "élémens des sciences", *Encyclopédie ou Dictionnaire raisonné des Sciences des Arts et des Métiers,* tome V (Paris: Briasson), 491-497.

Clairaut, Alexis-Claude (1741). *Éléments de Géométrie*

Clairaut, Alexis-Claude (1746). *Éléments d'Algèbre*. Paris: David l'ainé.

Davies, Charles (1839). *Elements of Geometry and Trigonometry*, translated from the French of A. M. Legendre, by David Brewster, revised and adapted to the course of mathematical instruction in the United States (New York: A. S. Barnes

Glaeser, Georges (1983). "À propos de la pédagogie de Clairaut", *Recherches en Didactique des Mathématiques, 4* : 332-344.

Knorr, Wilbur (1996). „The wrong text of Euclid. On Heiberg's text and its alternatives". Centaurus, *38,* 208-276.

Rommevaux, Sabine, Ahmed Djebbar & Bernard Vitrac (2001), *Remarques sur l'Histoire du Texte des Éléments d'Euclide,* Archive for the History of Exact Sciences, 55, 221-295.

Saito, Ken (2006). "A preliminary study in the critical assessment of diagrams in Greek mathematical works." *SCIAMVS*, 7, 81-144.

Saito, Ken (2007). "Les figures des Eléments dans les manuscrits et les éditions imprimées", Colloque « Traditions euclidiennes » 25-27 avril 2007, Clermont-Ferrand.

Schubring, Gert (1987). "On the methodology of analysing historical textbooks: Lacroix as textbook author", *for the learning of mathematics*, *7*, 41-51. ("Errata", ibid., 1988, *8*, 51).

Schubring, Gert (2003). *Análise Histórica de Livros de Matemática. Notas de Aula*. Campinas: Editora Autores Associados.

Schubring, Gert (2005). *Conflicts between Generalization, Rigor and Intuition. Number Concepts Underlying the Development of Analysis in 17th-19th Century France and Germany*. Sources and Studies in the History of Mathematics and Physical Sciences. New York: Springer.

Schubring, Gert (2007). „La diffusion internationale de la géométrie de Legendre: différentes visions des mathématiques", *Raisons – Comparaisons – Éducations. La Revue française d'éducation comparée*, *2*: 31-54.