Dialectic on the Problem Solving Approach:
Hermeneutic Efforts for Designing Classroom Mathematical Activity

Masami Isoda

Center for Research on International Cooperation in Educational Development, University of Tsukuba, Japan.

Project Overseer (Representative), APEC Lesson Study Project

Chief Editor, Journal of Japan Society of Mathematical Education JSME (since 1919)

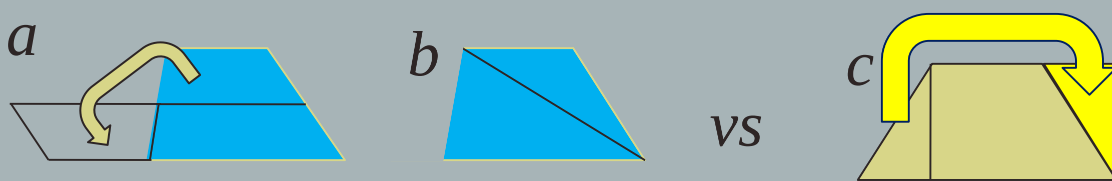
- ***Hermeneutics is the ground theory for interpretation. It's support understanding activity.***
- ***Hermeneutic efforts for designing classroom are the basic activity to develop theory for teaching.***

Content

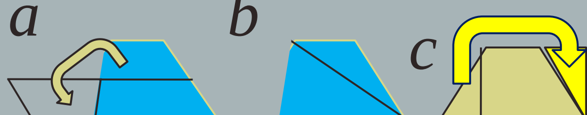
- *Objective: Knowing the importance of the hermeneutic efforts (ISODA, 2001)*
- *Setting*
  - *Knowing Problem Solving Approach*
  - *Knowing Hermeneutics (Abraham, Isoda, 2007)*
- *Examples*
  - *Internet Communication (Isoda, McCrae, Stacey 2007) for knowing the significance for humanizing mathematics.*
  - *Fraction (Isoda, 1993) for knowing the understanding beyond the cognitive veiw.*

Teaching Approaches which are developed through LS. Challengers try to develop new ways of teaching.

The lesson study community to make an effort to develop the children who learn…



No, some ideas cannot generalize



All ideas are fine.

Personal Level

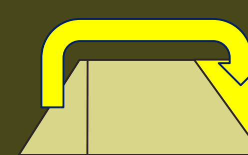
School Level LS

Dialectic

Problem Solving Approach

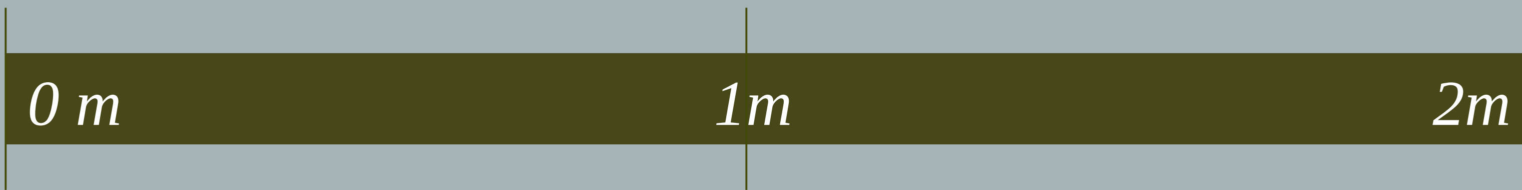
Open Approach

Injection

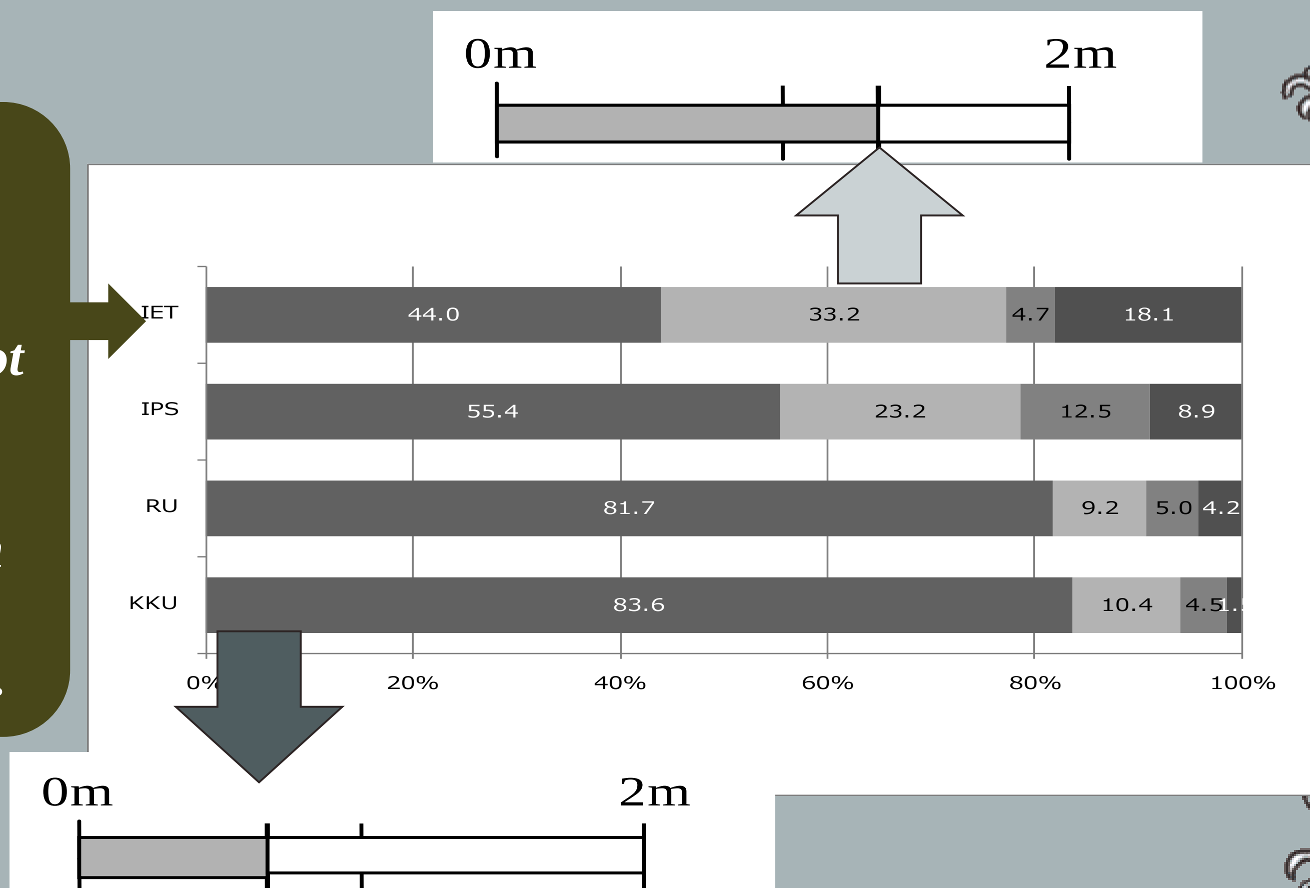


Where is the 2/3 m from the 2m tape?

By Isoda, Inprasitha, Anake: Nohda 1981



- ***The case of school teacher who do not know the lesson study with J. Textbook..***



Let’s see the video

- ***Why, the classroom became quiet.***
  - *Problem solving approach distinguish task and problem.*
  - *Why, some of them could get it and a few changed their understanding?*
- *How can you explain their understanding?*

Because they try to understand what Minami saying.

They can re-construct Minami’s idea by themselves.

In general

It can be said that because they try to get the others perspective.

We can get children’s perspective in Math Classroom.

We can get other teacher’s p. through the lesson study

What is Lesson Study?

In Isoda, M.(2011)
Journal of Science and Mathematics Education in Southeast Asia, v34 n1 p2-25

Process/Lesson Study cycle. Various dimensions of Open Classroom. Theme of Lesson Study. Lesson plan. Teachers' mind. Various Outcomes. Sequential experience for sharing the heritage. Additionally, developing children who learn by/for themselves

A US teacher said, I developed the eye to look at students and subject matters "Kodomo wo miru me." Now, I am well aware of my responsibility for my class. In the lesson study, with other teachers, I preferred the more challenging lessons such as with open-ended problems. When I found that students can challenge such difficult problems, I recognized self-confidence in my lesson. (Lewis, C. 2006)

Teachers are usually re-invent, discover or re-understand objective in relation to the children and subject matter through the lesson study and listening from others.

On this meaning, Lesson study is the reproductive science.

What is mathematics and it's learning?

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- ***Pre-established harmony***: *Unknown/not proved statement became the theorem in the system which is known.*
- *Mathematics learning should be done based on what learned before even if children have to extend their ideas.*

Constructivism: Organism VS Environment ← *Solipsism*

Mathematics is only existed in the mind?

Social Con.: Inter subjective to subjective ← *Materialism*

Mathematics is only existed in the language?

Through the activity for getting other's perspective we can bridge constructivism to social constructivism.

What is the theory of Education?

General vs. Local Theory in relation to the Classroom practice;

A Stereo Type images which does not have general meaning

- ***General Theory explains something.***
  - ***Observer observes, describes and say something but cannot propose.***
    - ***Epistemological Obstacle exists as obstacle but it does not tell how to beyond the obstacle itself.***
- ***Local Theory solves local situation.***
  - ***Local theory fixed on the situation is developed from ideas for challenges.***
  - ***Lesson study develops the local theory of teaching.***
    - ***Problem solving teaching approach tells how to beyond the obstacles.***
  - ***Local theory discuss necessary conditions for developing students but do not discuss sufficient conditions.***
- *General Theory in my research framework*
  - *Hermeneutics*
  - *Conceptual and Procedural Knowledge*
- *A theory for developing problem solving approach in classroom*
  - *Hermeneutic Efforts for developing mathematical knowledge with understanding.*
  - *Meaning and procedure for planning the lesson based on the curriculum which emended the children's conceptual development.*

Hermeneutics: General theory for Interpretation

- *General Theory:*
  - *Implicit theory: the methodology of qualitative study for scientific INTERPRETATION.*
  - *Explicit theory:*
    - *Modern hermeneutics: Gadamer, H.G. Hermeneutic cycle; "**we can not understand others**"*
    - *Historical hermeneutics: Schleiermacher, D. F. Hermeneutic cycle is the process for the subjective to objective; "**I can understand everything until contradiction.**"*
      - ***Until logical inconsistency, our interpretation, understanding, is true**.*
- *In Math Education:*
  - *Modern hermeneutics: Brown, T. (1997).*
  - *Historical hermeneutics: Jahnke, H.N. (1994)*
  - *Historian: Shubring, G. (2005)*

Hermeneutic Effort:
Originated between the lines:L on Hebrew

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Dead Sea Scrolls

ラメド

Hermeneutic Effort: local theory of understanding for humanizing mathematics education

Hermeneutic effort by Historians

First circle

Object

Scientist

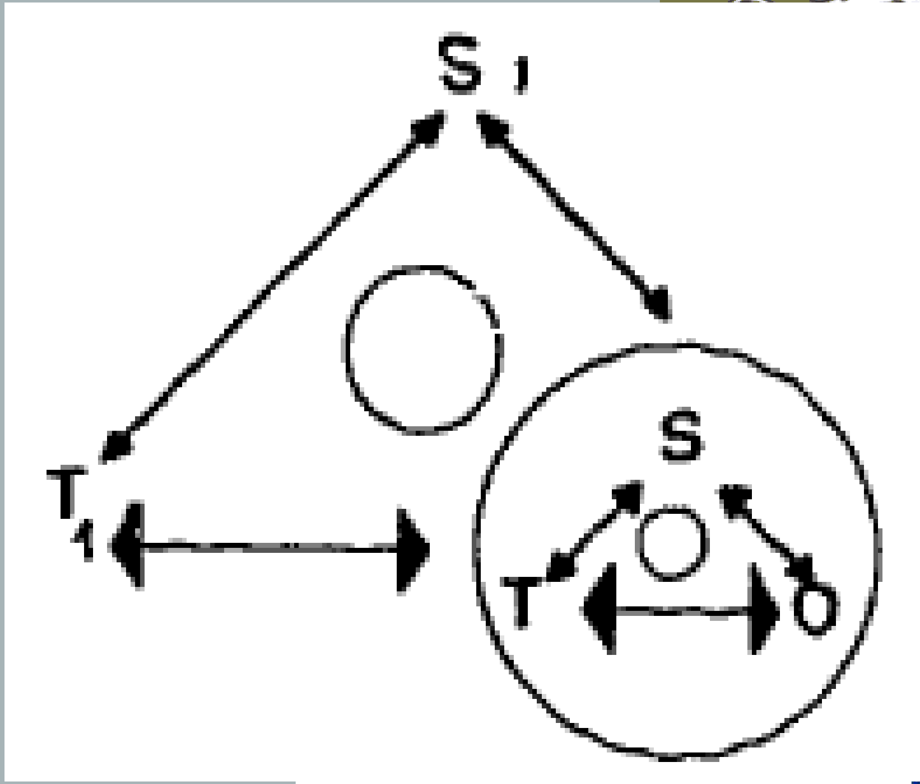
Theory

Second circle

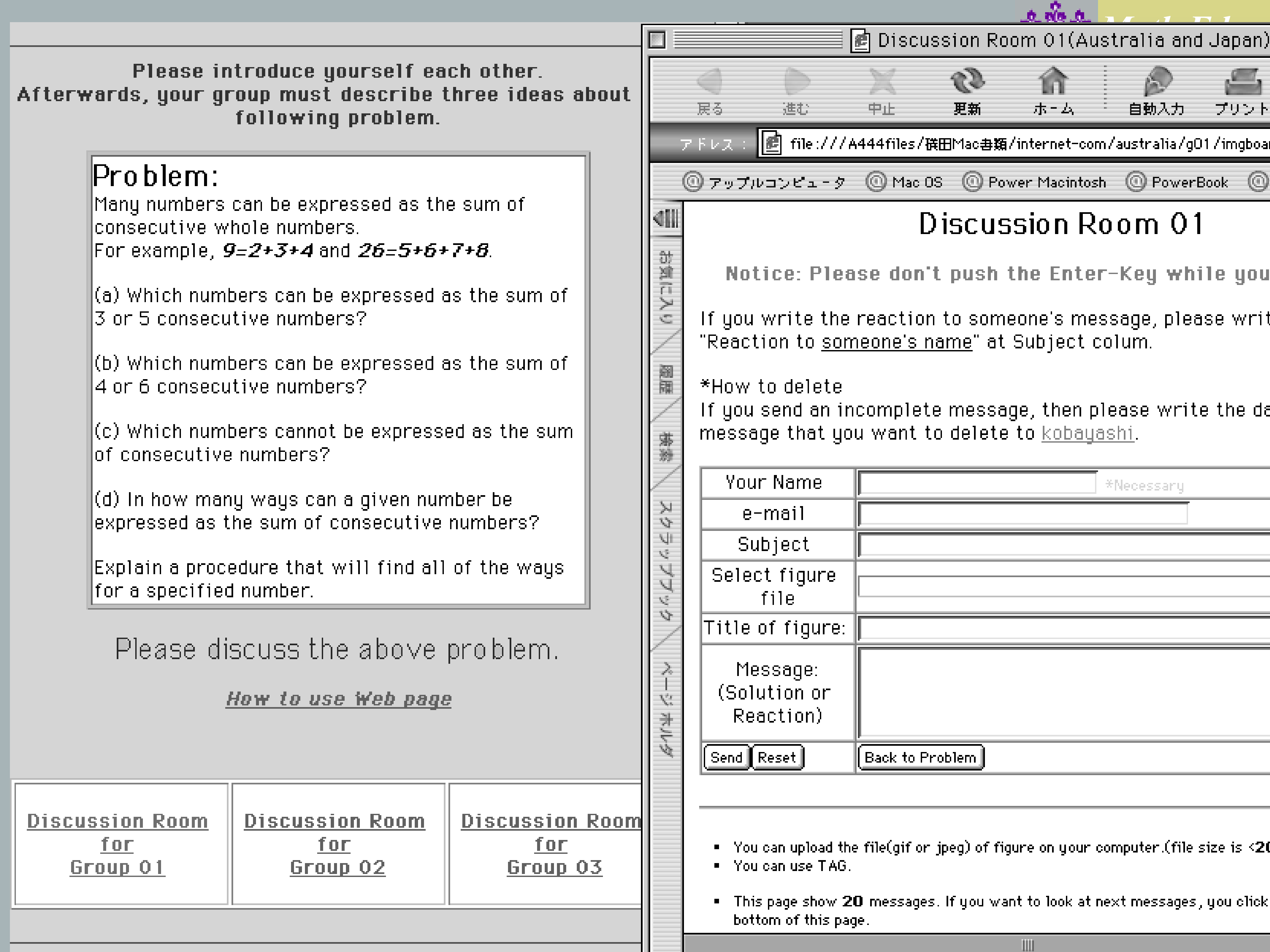
Object: past activity

S1:Historians

T1:Historical Interpretations



Jahnke, H.N. (1994)


Please introduce yourself each other.
Afterwards, your group must describe three ideas about following problem.
Problem:
Many numbers can be expressed as the sum of consecutive whole numbers.
For example, 9=2+3+4 and 26=5+6+7+8.
(a) Which numbers can be expressed as the sum of 3 or 5 consecutive numbers?
(b) Which numbers can be expressed as the sum of 4 or 6 consecutive numbers?
(c) Which numbers cannot be expressed as the sum of consecutive numbers?
(d) In how many ways can a given number be expressed as the sum of consecutive numbers?
Explain a procedure that will find all of the ways for a specified number.
Please discuss the above problem.
How to use Web page
Discussion Room for Group 01
Discussion Room for Group 02
Discussion Room for Group 03
Discussion Room 01(Australia and Japan)
戻る
進む
中止
更新
ホーム
自動入力
プリント
アドレス：
file:///A444files/internet-com/australia/g01/imgboar
アップルコンピュータ
Mac OS
Power Macintosh
PowerBook
お気に入り
履歴
検索
スクラップブック
ページホルダ
Discussion Room 01
Notice: Please don't push the Enter-Key while you
If you write the reaction to someone's message, please writ
"Reaction to someone's name" at Subject colum.
*How to delete
If you send an incomplete message, then please write the de
message that you want to delete to kobayashi.
Your Name
*Necessary
e-mail
Subject
Select figure file
Title of figure:
Message: (Solution or Reaction)
Send
Reset
Back to Problem
You can upload the file(gif or jpeg) of figure on your computer.(file size is <20
You can use TAG.
This page show 20 messages. If you want to look at next messages, you click
bottom of this page.

Hermeneutic Effort: local theory of understanding for humanizing mathematics Education

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Internet Project (Isoda, McCrae, Stacey 2007)

A Japanese Students said;

"It was fun that we could talk with students in far-off Australia. We could neither see them nor hear them, but the three of them certainly exist on the other side of the ocean and were thinking about the same mathematical problems as we were. Just imagining that makes me happy"

Imagine! The existence of others

Meet the Unknown

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☐ **Internet Project** name :P P [1999/10/26,08:51:49] *Australia*

(A) We have four members in our group. They are P P , E C , R t R and D d T . We are all between 14 and 15. We go to S in Melbourne, Australia. We all play *(OMISSION)*

(B) For part (a):
Let x be the first number, y be the second number and z be the third number in the consecutive sequence. The first sequence is 1,2,3 which adds to 6. The second sequence os 2,3,4 which adds to 9. The third possible values of x,y,z are 3,4,5 which add to 12. This is because the first sequence (1,2,3) can be represented by x,y,z. The second sequence will be (x+1),(y+1),(z+1) which is the same as x+y+z+3. The third sequence is (x+2),(y+2),(z+2), adding to x+y+z+3+3. The next equals x+y+z+3+3+3, and so on, adding 3 each time. So, starting at 6, they all go up in multiples of 3 when represented by consectutive numbers. The same method applies for adding with 5 consecutive numbers – it goes up in multiples of 5.

(C) Please reply to our suggestion for the answer of (a) and suggest spmething for part (b).

☐ **Reacton and our idea to solve (a) (b)** name :Y K [1999/10/30,22:20:51] *Japan*

(F) #As you know, we are not so good at writing English. So please let us know if you don't understand.

(OMISSION)

We read your message. The answer is same as ours. But we solve it in a differnt way. I think this way is easier than yours. You used three letters. But to use only X is easier. I'll show you our way.

---- part (a) -------

(G) #In this problem, we have to think 3 consecutive numbers and 5 consecutive numbers separately.

< 3 consecutive numbers >

Let the first number be X. As three numbers are consecutive, the next number must be (X+1).
In the same way, the last number must be (X+2). So the sum of these 3 numbers is...

X+(X+1)+(X+2)=3X+3 =3(X+1)

X will be natural number. (It can be taken for only integer which includes negative numbers.)

(OMISSION)

(H) Gentlemen and Gentlemen! (You are only boys) I hope you will understand the meaning of this expression.
Actually, when X=2 3(X+1)=9, when *(OMISSION)* Anyway the answer is <u>multiple of 3 bigger than or equal to 6</u>

Cultural Awareness
(Lerman, 1994. ISODA, 2001.)

- *Meet the Unknown.*
- *The Unknown Functions as the Mirror of Oneself.*
- *Cause for the Enculturation*

Japanese Students met

the sentence type description in Algebra, and so on.

Australian Students met

the formal type description in Algebra, and so on.

Cultural Awareness
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→ Our mathematics is more formal than Australian.

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→ Our mathematics is more basic than Japanese.

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< 5 consecetive numbers >

(OMISSION)

(I) #Question from us (1)

About "A" consecetive numbers. When "A" is an odd number, you can express the sum as multiple of "A".
When "A" is an even number, you can't express the sum as multiple of "A". Can you tell us why?

---- part (b) -------

#We considered part (b) in the same way. I'll show you waiting for your pointing out our mistakes.

< 4 consecetive numbers >

(OMISSION)

(J) The expression 2(2X+3) means that when X increases 1, the answer increases 2.
The answer is <u>multiple of 2 bigger than or equal to 12</u>

< 6 consecetive numbers >

(OMISSION)

#Question from us (2)

(K) We considered this problem over an basic condition. It is that the "numbers" means natural numbers.
But as I discribed before, "numbers" can be taken for integer which includes negative numbers.
If "numbers" means integer, how does the answer change?

---- Message ------

(L) Are you happy? Be happy! *(OMISSION)*

☐ **Internet Project** name :P   P   t [1999/11/03,07:30:13] *Australia*

(M) If negative numbers were included then the answer would be the same, but include all the answers as a negative as well as the positive.

part (b)

(N) The lowest number is 10 , this is because the numbers can be represented as (x, x+1, x+2, and x+3.) This is for the addition of 4 consectutive numbers.This works out as 4(x+1.5) As with your solution for part (a), It goes up in multiples of the amount of adding consecutive numbers, in this case, 4. This meanns the values are 10. 14. 18. 22. 26, etc...

For 6 numbers...

(OMISSION)

☐ **Let's think about part (c)!** name :Y   K   i [1999/11/05,21:05:34] *Japan*

(O) We read your letter. Your answer of #Question from us (2) was perfect!
If negative numbers are included, there is no minimum value. I think we discussed enough about (a) and (b).
But, have you discussed on #Question from us (1) in your group?
I will tell you the answer of it in the next letter. Please think about it again before the next letter comes.
Anyway, we want to go to part(c).

(OMISSION)

(P)

This chart means 1+2+3+4+5+6
0
0 0
0 0 0
0 0 0 0
0 0 0 0 0
0 0 0 0 0 0
It is similar to a right angled isosceles triangle.
Instead of counting all the points, calculate its area.

☐ **Internet Project** name :P   P   t [1999/11/08,

Discussion for part (c)

(Q) We like your idea, but we have another idea.
For this problem we will just focus on positive numbers, as you can get any numbers using negatives, eg. (−3)+(−2)+(−1)+(0)+(1)+(2)+(3)+(4)=8

(OMISSION)

Australia

Isoda, M. (2001) defined Hermeneutic effort for humanizing mathematics education by following activities.

Understanding; *one's interpretation*

Getting others' perspectives (the assumption of the positions of others); *the appropriate interpretations of a text is only possible through* ***a subjective approach whereby we assume*** *the writer's (or speaker's) position, feelings and sympathetically attempt to put ourselves into the position of another (writer or speaker).*

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Isoda, M. (2001) defined Hermeneutic effort for humanizing mathematics education by following activities.

Instruction from experience (self-understanding)；*one obtains an instruction about oneself* ***with comparison of others' perspectives.***

The hermeneutic cycle: *if we have some understanding, we apply it to new situations and if it is applicable, it will become more objectively correct.*

'Hermeneutic cycle'='subjective to objective'

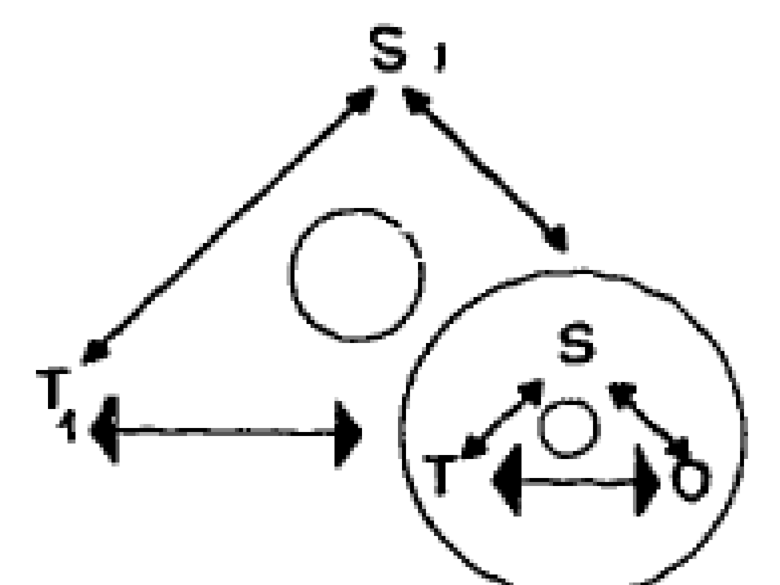
Case Study for Illustrating H.E.

Divisional/Partitive Fractions vs. Quantitative Fractions

How difficult for ***getting other's perspectives****.*

What kinds of arguments will be necessary.

Counter examples are not always counter examples.

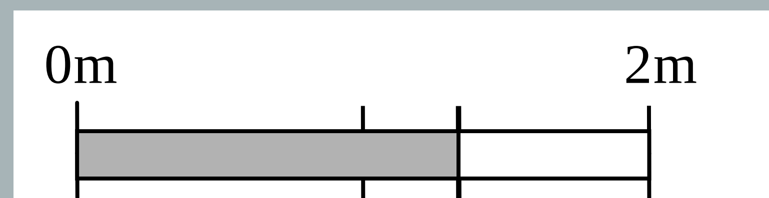


Making (creating) a 2/3 m piece of tape from a 2 m piece of tape;

even if they already learned.

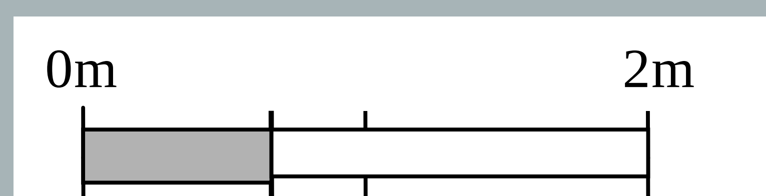
- **Partitive/divisional fractions;** *fraction in partition, n parts from among m equally divided parts of the whole*

  37 children



- **Quantitative fractions;** *n parts from among m equally divided parts of a unit quantity (such as '1 m'), where m<n is also possible such as '3/2 m'*

  1 children

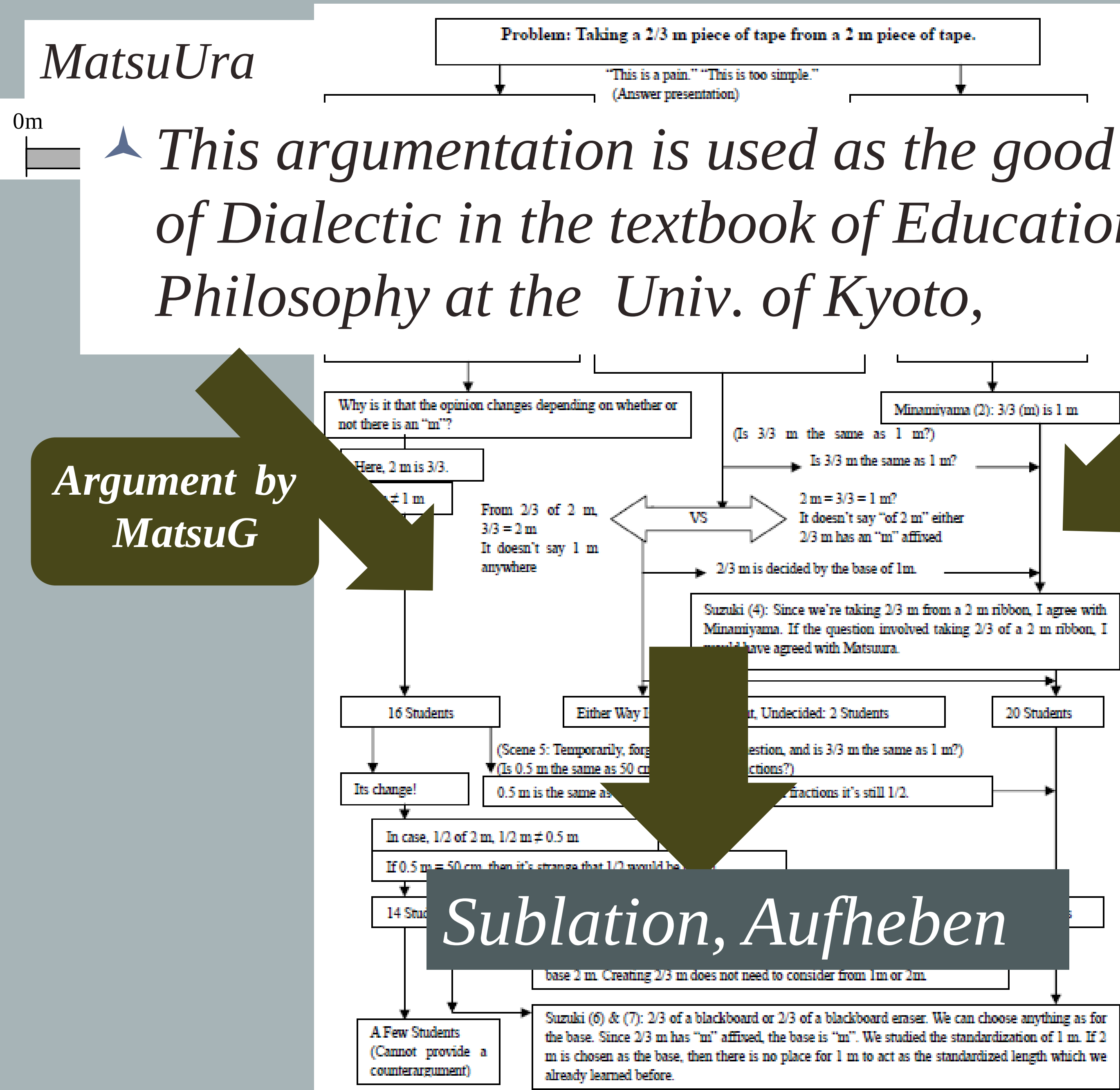


Dialectic between two ideas

MatsuUra

MinamiYama

- *This argumentation is used as the good example of Dialectic in the textbook of Educational Philosophy at the Univ. of Kyoto,*



Argument by MatsuG

Argument by MinamiG

Sublation, Aufheben

Scene 1: Presentation of the Results of Individual Problem-Solving

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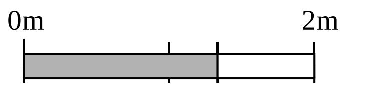
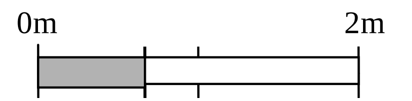


MatsuUra; *37 students*



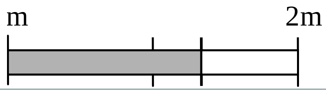
MinamiYama; *2 students*

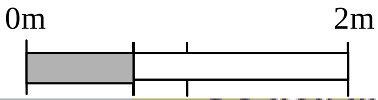
MinamiY (1):	Basically, it's half of this (MatsuUra's tape).
MatsuU G:	No, it's two thirds!
MatsuU G:	Wait, I get it; I'm in MinamiYama's Group, too.
MatsuU G:	The of 2
Suzuki (1):	Tea

When MatsuU G knew the MinamiY idea, some of them remembered what they had learned before. MinamiY idea functioned as a counter example.

Matsuura Group: two parts of the three equal segments of the 2 m	Either Way Is Fine	Undecided	Minamiyama Group: 2/3m
22 students	11 students	2 students	4 students

Scene 2: Exchanges Regarding Each Side's Position (Each others' Opinion)

MatsuUra; 

MinamiYama 

Either G: Maybe the problem is coming from the way of question. According to Matsuura Group, the answer is 2/3 of 2 m.

MinamiY G: But it says "from 2 m."

Either G: So the reason for Matsuura's answer of 2/3 is that since 1 or 1 m is taken from the 2 m tape.

MatsuU G: You just divide it into three and take two of those segments.

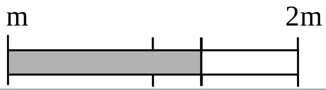
MinamiY G: But it's 2/3 from the 2 m tape

(Note: the "m" of "2/3 m" is missing in his explanation).

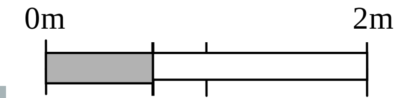
Either G understand both sides

Scene 2: Exchanges Regarding Each Side's Position (Each others' Opinion)

MatsuUra;



MinamiYama



Suzuki (2): This is a 2 m piece of tape, so with 2 m, you get 2/3 m, right? Usually when you have a fraction, the base number is 1. Since it's 2/3 m here, you have to get the base to 1. It says 2/3 m, right? Since there's an "m" on it, that means 2/3 of 1 m. So it's 2/3 m from 2 m of tape, and Minamiyama first threw out this half (1 m), and I think you use two of the three segm Matsuura's Group did this wit would be just like Matsuura's answer.

Suzuki understand both sides and explain correct answer.

MinamiY G: Now there's the "m", so wouldn't Minamiyama be right?

MatsuU G: Why is it that Matsuura's right **only** if there is no "m" (in "2/3m")?

MinamiY(2): I thought that 3/3 is equal to 1 m.

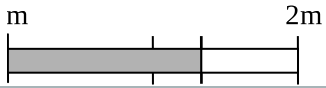
Teacher (1): One more time.

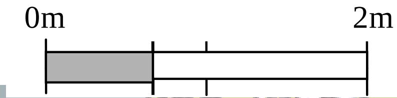
MinamiY(3): 3/3 means 1 m, right?

(Note: the "m" of "3/3 m" is missing in his explanation).

MatsuU G do not understand Suzuki's explanation .

Scene 3: Teacher Intervention 1

MatsuUra; 

MinamiYama 

Teacher (2): 3/3 m is 1 m, no doubt about it (Note: he emphasized the "m").

MinamiY G: Right!

MatsuU G: No, absolutely not.

MatsuU: They already feel contraction but reasoning from conclusion

Suzuki (3): Teacher, it is not related with the problem (Note: the original question), isn't it?

MatsuU G: You take 2/3 from 2 r … n.

MinamiY(4): (Pointing at the 2 m f … are 2/3.

Suzuki try to share the ground of reasoning.

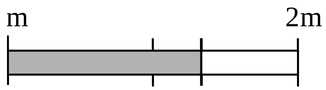
MatsuU G: No mentioned 2/3 of "1 m" in the original problem.

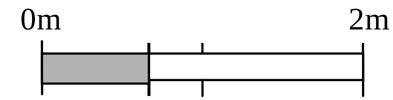
MinamiY G: It doesn't say create "2 m" tape. It just says is "from 2 m of tape".

MinamiY G: Since the original problem doesn't say to make this only from a 2 m tape, you can make it from 1 m as well.

Teacher (3): Minamiyama, if your answer is 2/3 m, then we would like to say that the base is 1 m. This is the reason why 3/3 m is 1 m, and the base is 1 m, according to what you are trying to say, right, Minamiyama?

Scene 3-4: Intervention-Sharing

MastuUra; 

MinamiYama 

Teacher (4): Well, this is a probler

MatsuU G: Since Minamiyama h
remain 1 and 2/3?

MatsuU G develop their meaning of MinamiY on their ground,

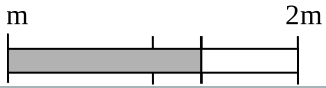
MatsuU G: So Minamiyama does not take 1, but 1/6.

Teacher (5): No. Minamiyama’s answer works when he’s only using this (1 m). The remaining 1 m is irrelevant for him.

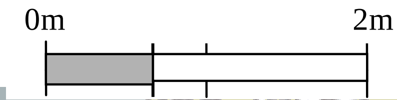
Suzuki (4): If the original problem involves making 2/3 of a 2 m tape, then Matsuura’s side is right, I mean, I think Matsuura’s argument is easier to understand. Since you’re supposed to create 2/3 m from a 2 m piece of tape then it must be 2/3 m. So you ignore the 1 m, and this 3/3 m is also 1 m. Since you are going “from”, you’ve got to deal with both “from” and “m”. If there wasn’t this “m”, and if “from” was “of”, then I would agree with Matsuura. (Repeating while reviewing the figure) This 2/3 m means that the base is 1 m. If there wasn’t an “m”, then you could use any amount of “m” as the base, but since there is an “m”, then 1 m must be the base.

Scene 4: Sharing

MatsuUra;



MinamiYama



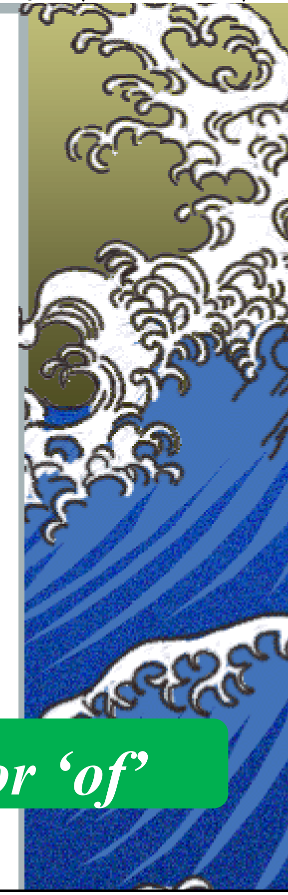
MatsuU G: If the problem is “create 2/3 from a 2 m tape”, or “create 2/3 m of a 2 m tape”?

MatsuU G & MinamiY G: The first one, “create 2/3 from a 2 m tape”, is Matsuura Group but what about the second one?

MatsuU G: 2 m might be the base, but since its 2/3 m, 1 m might be the base, too.

Teacher (6): So the second one would be strange and contradicting.

Shared the Problematic ‘from’ or ‘of’



Matsuura Group: two parts of the three equal segments of the 2 m	Either Way Is Fine	Undecided	Minamiyama Group: 2/3m
16 students	No student	2 students	20 students

Scene 5: Teacher Intervention 2

- *In order to stir things up again, the teacher asked the students to forget the original question and whether or not "3/3 m is 1 m" temporarily. Some of Matsuura group are still the opinion that "it is three equally divided parts of 1 m or 2 m". This opinion indicates that some students are still caught up in the idea of divisional fractions.*
- *The teacher asks "can we change tracks?" and continued as follows.*

Scene 6: Teacher Intervention 3

Teacher (7): If we have 0.5 m, then do we indicate what the length is in cm?

MatsuU G &MinamiY G: Yes, it's the same as 50 cm.

Teacher (8): Can we express this as a fraction? (Detailed discussion omitted) So is it the same as 1/2 m, or is it different?

Some MU understand but some developed hard core.

MinamiY G: It's the same.

MatsuU G: Wow!

(Note: this is taken to mean that they are realizing their contradiction.)

MatsuU G: It's different.

Suzuki (5): If 0.5 m is the same as 1/2, then what is 1/2?

Teacher (9): And if I asked you to express 1/2 m as a decimal of m, what would that be? (Note: he added 'm'.)

MatsuU G: 0.5. (Note: it still lost 'm'.)

MatsuU G: It might be 1/2 of 2 m.

MinamiY (5): 0.5 m = 1/2 m and 1/2 m = 0.5 m are the same thing, all you're doing is reversing the order. So I think you can say that 3/3 m is 1 m. But if the base changes, I'm not sure if you can still say that 1/2 m = 0.5 m.

Matsuura Group: two parts of the three equal segments of the 2 m	Either Way Is Fine	Undecided	Minamiyama Group: 2/3m
14 students	No student	1 students	23 students

Scene 7: The Next day

After some feedback.

Teacher (10): The class seems to be in Minamiyama's direction. Matsuura Group, do you have anything to add to the discussion?

MatsuU G: It says "from" a 2 m tape, right? If it said "from a 1 m tape", or if it didn't say "from" ("of 1m"), then Minamiyama would be right, but it does say "from", so 2 m is the base.

MinamiY G: 2 m is larger than 1 m, right? So we can just forget 1 m of the 2 m for the moment, and take 2/3 m from 1 m, for instance.

MinamiY G: Just ignore where it says "from".

Teacher (11): So that you are saying, just "create a 2/3 m tape" is the same as the original question.

Suzuki (6): For instance, you have a blackboard and you have 2/3 of a blackboard. We say this is 2/3. For instance, if you have a blackboard eraser, you could say 2/3 of this blackboard eraser. Understand?

Teacher (12): I know what you're trying to say. I really do understand.

Scene 7: The Next day

Suzuki (7): You can go with anything whatever. But it says 2/3 m. Since it has an "m" on it, that "m" must be the base. We studied that it was determined by the distance from the Equator to the North Pole divided by some tens of millions, right? Before they standardized it that way, "1 m" was not always equal, right? If you use 2 m as the base, you back then against the determination. Anyway, since m has 1 m as the base. This is the difference between when you have a given base and when you don't.

Suzuki try to share the ground.

Teacher (13): Suzuki wants to say that since there is a unit affixed, the base is already completely settled. So that's why she feels she has to join the Minamiyama Group.

Teacher (14): We haven't heard from the Matsuura Group at all lately. Can we end this discussion now, then? Since "m" is affixed, the base is "1 m", but since we are dealing with 2/3, we can change the base accordingly. It's as simple as that, isn't it? Is this fine with everyone? (The class ends at this point.)

Although there are no longer any counterarguments from Matsuura Group, some of the members remain unconvinced.

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Problem: Taking a 2/3 m piece of tape from a 2 m piece of tape.

"This is a pain." "This is too simple."
(Answer presentation)

Matsuura Group: Majority of students.
Cuts the tape into two parts of the three equal segments of the 2 m
[Meaning a divisional fraction]

VS

Minamiyama Group: Two Students
2/3 m (1/3 of 2 m)
[Meaning a quantitative fraction]

No, it's two thirds! Wait, I get it
I'm in Minamiyama Group, too.
(The problem is within the question!)

22 Students

Either Way Is Fine: 11 Students, Undecided: 2 Students

4 Students

MatsuUra

0m 2m

MinamiYama

0m 2m

learned?

lost 'm'

Either G understand both sides

Why is it that the opinion changes depending on whether or

MatsuU G do not understand.

Suzuki understand both sides and explain correct answer.

3/3 m ≠ 1 m

From 2/3 of 2 m

2 m = 3/3 = 1 m?

Reasoning from conclusion

3/3 m =1m?

Suzuki try to share the ground.

Shared the Problematic 'from' or 'of'

0.5m ?

16 St

2 Students

20 Students

(Scene 5: Temporarily, forget the original question, and is 3/3 m the same as 1 m?)
(Is 0.5 m the same as 50 cm? What about fractions?)

hange!

0.5 m is the same as 50 cm, and even in the case of fractions it's still 1/2.

In case, 1/2 of 2 m, 1/2 m ≠ 0.5 m

If 0.5 m = 50 cm, then it's strange th

MatsuU G reason on their ground and developing hard core.

14 Students

Eithe

Teacher support Suzuki

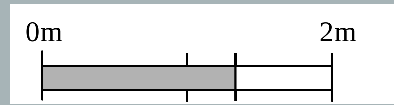
If the m unit is affixed to 2/3,
base 2 m. Creating 2/3 m doe

Suzuki assert the invariant of 1m.

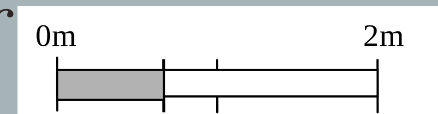
Few Students
(Cannot provide a counterargument)

Suzuki (6) & (7): 2/3 of a bla
the base. Since 2/3 m has "m" affixed, the base is "m". We studied the standardization of 1 m. If 2 m is chosen as the base, then there is no place for 1 m to act as the standardized length which we already learned before.

Types of Counter Example for MatsuU



Type A: MinamiY's answerer itself



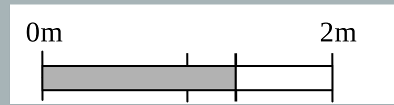
Type B: 3/3 m=1m (not functioning well because it deny MatsuU)

Type C: 0.5m=50cm (Possibility of Generalization of MatsuU; some of them functioning)

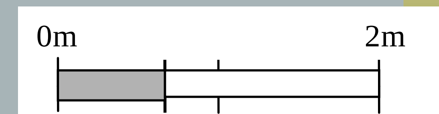
Suzuki's equator is related with Type C.

Type B & C developed the hard core at the same time.

Why it’s fucntioned as counter example?



Type A: MinamiY’s answerer itself



Type B: 3/3 m=1m (Even if feeling strange, not functioning well because it deny MatsuU)

Type B’: 0.5m=50cm (Possibility of Generalization of MatsuU; some of them functioning). If we deny it, 1m is not invariant (Suzuki’s last comment)

Type B & B’ developed the hard core, too.

Teacher's roles

First, he gave opportunity of assertion from both sides (Scene 1),

Second, he allowed Suzuki who understands both sides to explain,

Third, he gave a counter example 3/3m in relation to the original question (Scene 3),

Fourth, he gave generalized counter example 1/2m (Scene 6), and

Fifth, he supported Suzuki who explained the necessity to fix '1m' as a unit for conclusion.

Why teacher have to support Suzuki?

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Why some children developed hard core?

Because they try to say their conclusion is true.

Because they do not well understand.

Because they thought emotionally but did not logically.

In the process, they applied	Type A	Type B	Type C
the appropriate procedure of division	Kept	Kept	Kept
the appropriate meaning of the quantity	Kept	non → having	non → non

Case Study for Illustrating H.E.

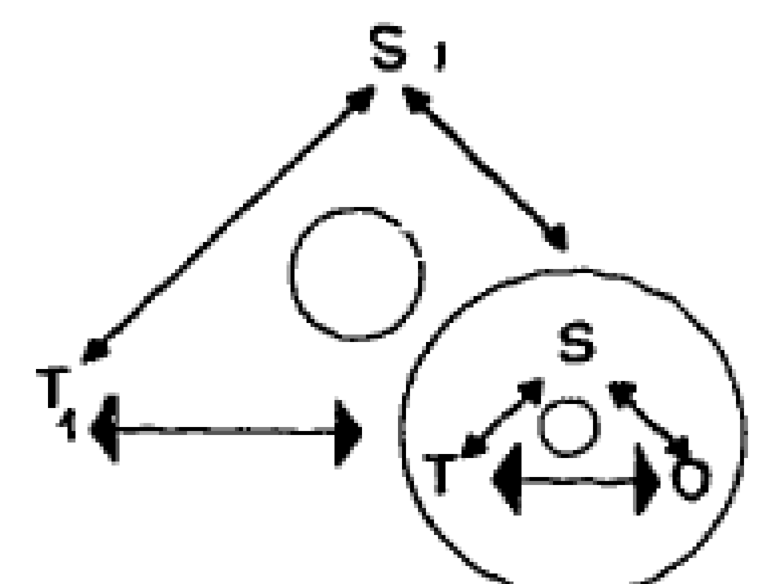
Divisional (Partitive) Fractions vs. Quantitative Fractions

*How difficult for **getting other's perspectives**.*

What kinds of arguments will be necessary.

Counter example is not always counter example.

Depending on the teaching sequence we can avoid the developing hard core!



Conclusion

- *Objective: Knowing the importance of the hermeneutic efforts (ISODA, 2001)*
- *Setting*
  - *Knowing Problem Solving Approach*
  - *Knowing Hermeneutics (Abraham, Isoda, 2007)*
- *Examples*
  - *Internet Communication (Isoda, McCrae, Stacey 2007) for knowing the significance for humanizing mathematics.*
  - *Fraction (Isoda, 1993) for knowing the understanding beyond the cognitive veiw.*