Mathematics and Mathematics Education: the technology as strategy of a hermeneutical approach of the History of Mathematics to teach school mathematics content

Tsukuba Global Science Week

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Session *"Hermeneutics in Mathematics Education: History of Mathematics to Imagine the Future and Understand the Perspective of Others*

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Summary

- Reflections from the perspective of Teacher Educator and some projects for the Professional Development of Teachers:
  - **Mathematics**:
    - Evolution of mathematics
    - The importance of the **History of Mathematics** to understand the evolution of mathematical concepts and their essence through the time;
  - **Mathematics Education**:
    - The importance of the **History of Mathematics** in the education of teachers: to support/improve the teaching and to promote the learning (what, when, how)
  - **The Hermeneutics** as grounding principle in developing teaching materials for the professional development of school teachers:
    - **Technology** of educational software as strategy to improve the interpretation of mathematics concepts through historical texts, *to understand* the mathematic activities for the learning, *to perceive* the perspective of the others for a better assesment.

Mathematics

- There is strong need to desmistify a common understanding (at least for school teachers and general people) that Mathematics has been organized since ancient times as it appears in the school textbooks, as exact and abstract science with known rules and calculation methods.
  - What is the meaning of the historical discoveries and accomplishments that accompanied the civilizations and societies?
- History of Mathematics is a way to understand the genesis and the evolution of mathematics as a human enterprise through the time;
- Utmost important questions are raised:
  - Then, what are the meanings of mathematics curriculum at basic schools in the teacher education courses?
  - How could we appreciate the deep implication of mathematics discoveries in the contemporary world of changes?

Mathematics Education

- The challenge of understanding the meaning of mathematics in the school curriculum of basic education.
  - What do the Teacher Education courses need to entail further professional development to improve the practice?
- The realization of the importance of knowing the historical evolution of concepts to understand the difficulties of learning as well as the semiotic meaning of instruments for learning.
  - Objective: Update the teaching methodologies to attend the necessary paradigm shift in classroom practices, to master the lesson planning to develop competencies in meaningful **Problem Solving (mathematics activity).**

How can teacher educators prepare and support teachers prepared to this scenario? Where can the teachers look for the inspiration to get insightful vision of Mathematics as accumulated scientific knowledge, and at same time, a school content knowledge suited to the education of citizens?

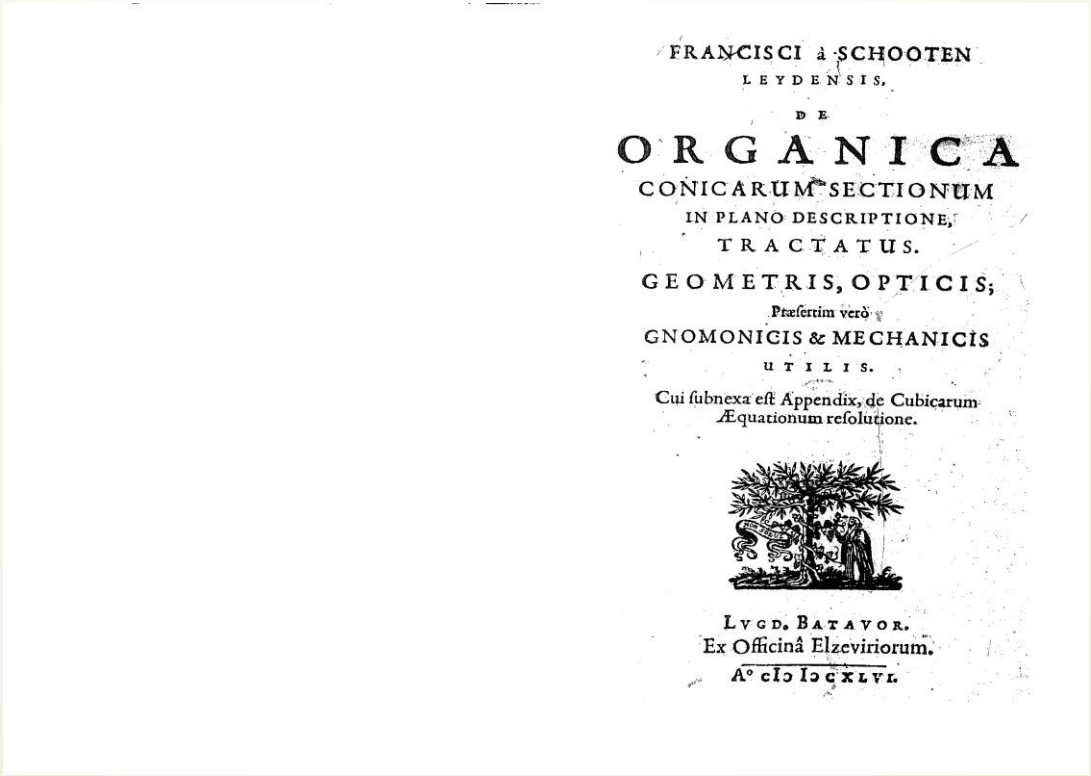
Hemeneutics in Mathematics Education

- The hermeneutics effort as focused in this Session:
  - a conceptualization of hermeneutics as the theory of interpretation, that is, a theory of achieving an understanding of texts; (Forster)
  - Moreover, narrowing into a restricted focus, as a theory of understanding for planning the problem solving approach; (Isoda)
  - Activity of the recursive process of interpretation to explain the role of history for education, in order to educate pupils and teachers to think oneself into another person who have lived in another time and another culture.... (Jahnke)
  - To understand the perspective of others (Isoda)

Technology as strategy of hemeneutical approach to mathematics content, school curriculum topics and methodologies of teaching that connect the meanings of different topics, developed with historical perspective.

Two exemplary projects in Mathematics and Mathematics Education on Hermeneutics of Historical texts, with the aid of technology: M. Isoda et al.

- **D-book software to study and to interpret the famous Schooten´s book (17th century) on Conic Sections in Teacher education courses at U. Tsukuba and Workshops abroad (in BRAZIL: Lesson Study at UFRJ and USP)**


FRANCISCI à SCHOOTEN
LEYDENSIS,
DE
ORGANICA
CONICARUM SECTIONUM
IN PLANO DESCRIPTIONE,
TRACTATUS.
GEOMETRIS, OPTICIS;
Præsertim verò
GNOMONICIS & MECHANICIS
UTILIS.
Cui subnexa est Appendix, de Cubicarum
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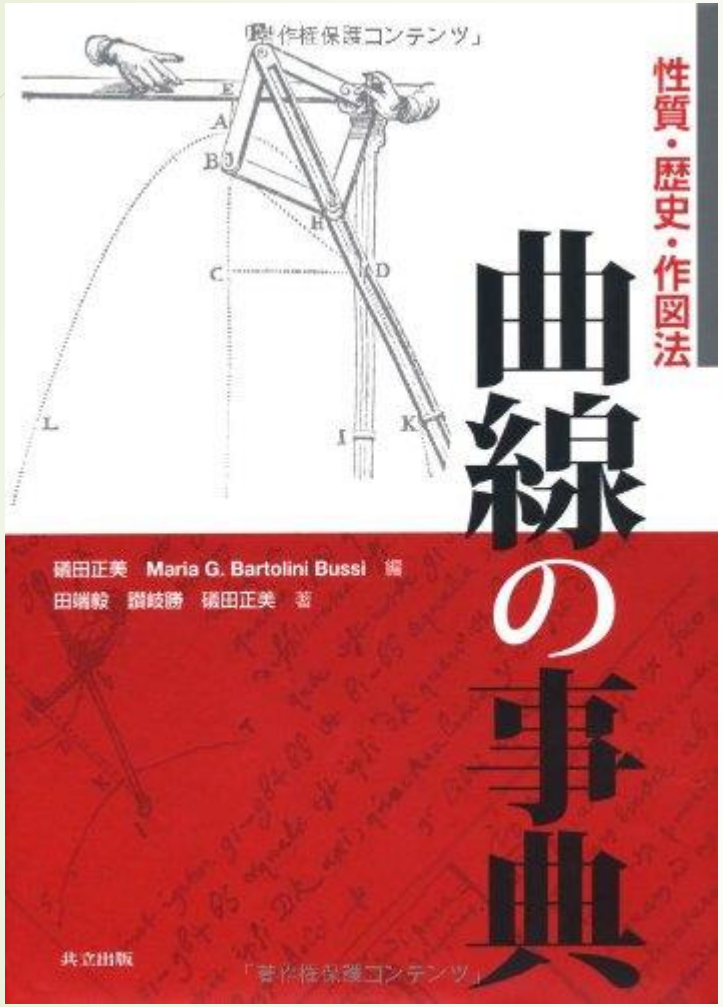


AWARD WINNING PUBLICATION
(ISODA &BUSSI, 2009)
ENCYCLOPEDIA OF CURVES, TOKYO,
Kyouritsu sha

Examples from the research in Teacher Education and Professional Development Projects (Baldin)

- Technology as strategy of hermeneutical approach to History of Mathematics in:
  - Research activities of preservice teacher
  - Didactical material for teacher education courses (pre-service and in-service)
  - Design of teaching material for classroom use at basic education level

Example1: Research Work of a preservice teacher (HASEGAWA, 2006) on History of Mathematics.
Providing a visually concrete model to retrieve/interpret the original 3-dimensional definition of conic sections, following Appolonius.

A GEOMETRIA DO *LATUS RECTUM* NO ESTUDO DE CÔNICAS SEGUNDO APOLÔNIO DE PERGA

Ademir Cristovão Lucchiari (IC); Yuriko Yamamoto Baldin (O) – DM-UFSCAR

INTRODUÇÃO: Mesmo conhecido anteriormente aos trabalhos de Apolônio, desde Menaecmus (séc. IV – V A.C.) considerado o descobridor das secções cônicas, o ***latus rectum*** (ou parâmetro de uma cônica) foi geometrizado na obra *As Cônicas*, por Apolônio de Perga (262 a 190 A.C. aproximadamente).

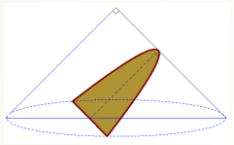
Este trabalho busca uma interpretação das proposições de Apolônio que definem as secções cônicas numa linguagem matemática moderna, e demonstra suas construções utilizando régua e compasso. Também traz uma visão bidimensional dessas construções por meio do rebatimento do espaço para um plano, facilitando o entendimento.

Através da tradução e interpretação da obra de Apolônio a partir da versão inglesa (On Conics Sections, in Great Books of the Western World, Robert Maynard Hutchins (ed), Encyclopaedia Britannica, 22nd ed, 1978, Vol. II), e com a utilização do software Cabri Géomètre II, demonstramos como as três secções cônicas eram conhecidas e obtidas, como o *latus rectum* era construído e como veio a determinar os nomes das secções, permanecendo até os dias atuais.

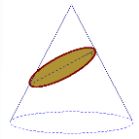
PALAVRAS-CHAVE: Secções cônicas, latus rectum, uso de tecnologia no ensino.

Visão antes de Apolônio

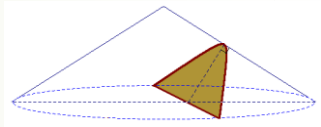
As secções eram obtidas através do corte de um cone circular reto por um plano perpendicular a uma reta geratriz do cone. Cones com um ângulo reto (cone retângulo), menor (cone acutângulo) ou maior (cone obtusângulo) em seu vértice geravam, respectivamente, um *orthotome* (hoje parábola), um *oxytome* (hoje elipse) ou um *amblytome* (hoje hipérbole).



Orthotom



Oxytome



Amblytom

Visão de Apolônio

Apolônio passa a utilizar um cone único para obtenção das três secções cônicas, variando a inclinação do plano que o corta, sem necessidade da perpendicularidade relativa a uma geratriz. Também introduz o cone de duas folhas e o cone obliquo, em substituição ao cone reto.

O *Latus Rectum* $\left(l = \frac{AS.BC^2}{AB^2}\right)$

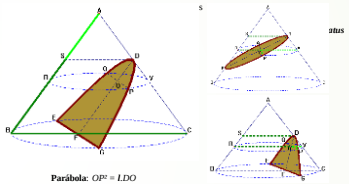
(1) $RO = SD$ (lados do paralelogramo); (2) $OP^2 = RO.OV$ (média geométrica);

(3) $\frac{OV}{DO} = \frac{BC}{AB} \Rightarrow OV = DO.\frac{BC}{AB}$ $\quad \frac{SD}{AS} = \frac{BC}{AB} \Rightarrow SD = AS.\frac{BC}{AB}$

e

$OP^2 = SD.OV = AS.\frac{BC}{AB}.DO.\frac{BC}{AB} = \frac{AS.BC^2}{AB^2}.DO$

Então, $\frac{AS.BC^2}{AB^2}$



Elipse: $RO < SD \rightarrow OP^2 < SD.OV.$ Então $OP^2 < LDO$

Hipérbole: $RO > SD \rightarrow OP^2 > SD.OV.$ Então $OP^2 > LDO$

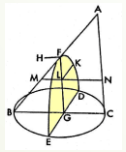
Parábola: $OP^2 = LDO$

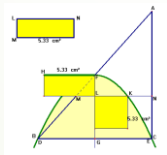
A parábola

Conforme proposto por Apolônio, a parábola é definida a partir do segmento *FH* (*latus rectum*), onde $\frac{BC^2}{BA.AC} = \frac{FH}{FA}$. Na figura, $KL^2 = LF.FH$ para qualquer *K* pertencente à secção cônica.

Na figura seguinte rebatemos a parábola para o plano do triângulo *ABC* ($BC = DE$) e vemos a igualdade de áreas entre KL^2 e $LF.FH$.

Considerando um referencial ortogonal, com a reta suporte do segmento *KL* = eixo *y* e a reta suporte do segmento *LF* = eixo *x*, $KL^2 = LF.FH$ seria $y^2 = lx$, onde *l* é o *latus rectum*.





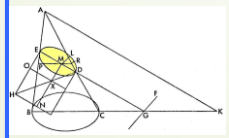
A elipse

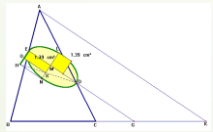
Para a elipse, com *latus rectum EH*, temos $\frac{AK^2}{BK.CK} = \frac{DE}{EH}$

Na figura, $LM^2 = EM.MX < EM.EH$, para qualquer *L* na secção cônica.

Na figura seguinte (rebatimento) vemos a igualdade entre as áreas, $LM^2 = EM.MX$.

Considerando o referencial ortogonal acima, com $LM = y$ e $EM = x$, temos $y^2 < lx$.



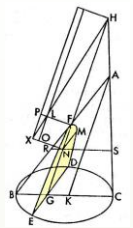


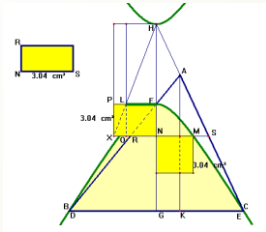
A hipérbole

Para a hipérbole, assim como nos casos anteriores, vemos o *latus rectum FL*, onde $\frac{AK^2}{BK.KC} = \frac{FH}{FL}$ e $MN^2 = FN.NX > FN.FL$, para qualquer *M* na secção cônica.

Na figura seguinte vemos o rebatimento com a igualdade entre as áreas, $MN^2 = FN.NX$.

Considerando o mesmo referencial ortogonal, com $MN = y$ e $FN = x$, temos $y^2 > lx$.





Como pudemos observar, a álgebra dos gregos era totalmente baseada na geometria. As demonstrações das construções das secções cônicas saem de comparações e razões entre segmentos, entre áreas e entre segmentos com áreas, tratando uma figura espacial como algo que pudesse ser geometrizado no plano. Apesar de não utilizarem um sistema de coordenadas cartesianas, como entendemos hoje, parece-nos evidente que trabalhavam com um sistema ortogonal, mesmo que implicitamente.

A comparação de y^2 com lx, sendo igual, menor ou maior, veio a definir o nome das curvas substituindo os nomes utilizados até então e permanecendo até os dias atuais. O nome *parábola*, indicando colocar ao lado ou comparação, substituiu o nome *orthotome*; o nome *elipse*, significando falta de alguma coisa, substituiu *oxytome*; e o nome *hipérbole*, significando um lançamento além, ou um excesso, substituiu o nome *amblytome*.

CONCLUSÃO: O presente trabalho é uma contribuição para o ensino e aprendizagem da geometria, com um material que serve como apoio a professores e estudantes de ensino médio e das licenciaturas,sobre um tópico que está relegado a um plano secundário, a despeito da sua importância na matemática.

Example 2: A RESEARCH WORK BYA PRESERVICE TEACHER ON THE CONCEPT ***OF LATUS RECTUM*** IN THE ENGLISH TRANSLATION OF **THE ORIGINAL "ON CONICAL SECTIONS"**, BY APPOLONIUS, WITH THE USE OF TECHNOLOGY.

HERMENEUTICAL EFFORT TO UNDERSTAND THE CONCEPTUAL GENESIS AND TO RELATE TO MODERN PRESENTATION OF CONICS.

FOR THE BETTER KNOWLEDGE OF TEACHERS!

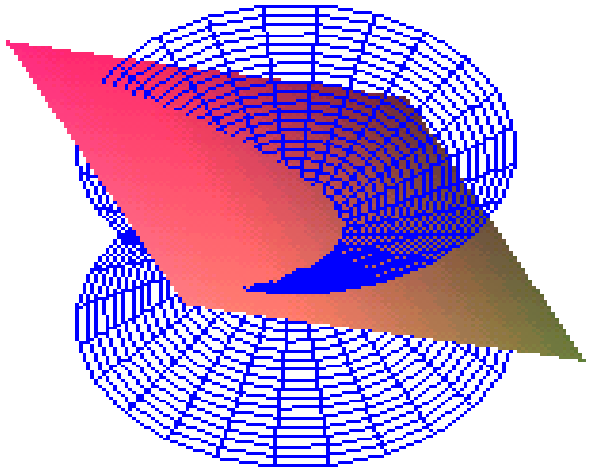
For the knowledge of teachers: didactical materials developed for teacher education

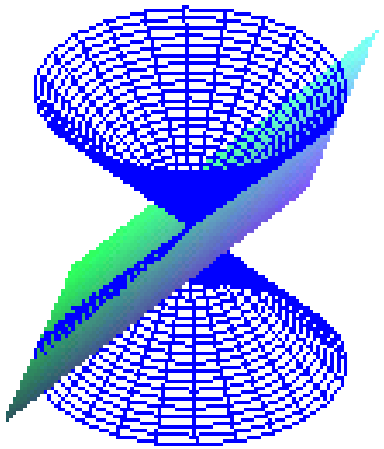
- Hermeneutical approach to the Conics following the conceptions of Appolonius of Perga , 3-dimensional setting explored with CAS-technology dynamical features to enhance the interactivity. Visualization to interpret the concepts.

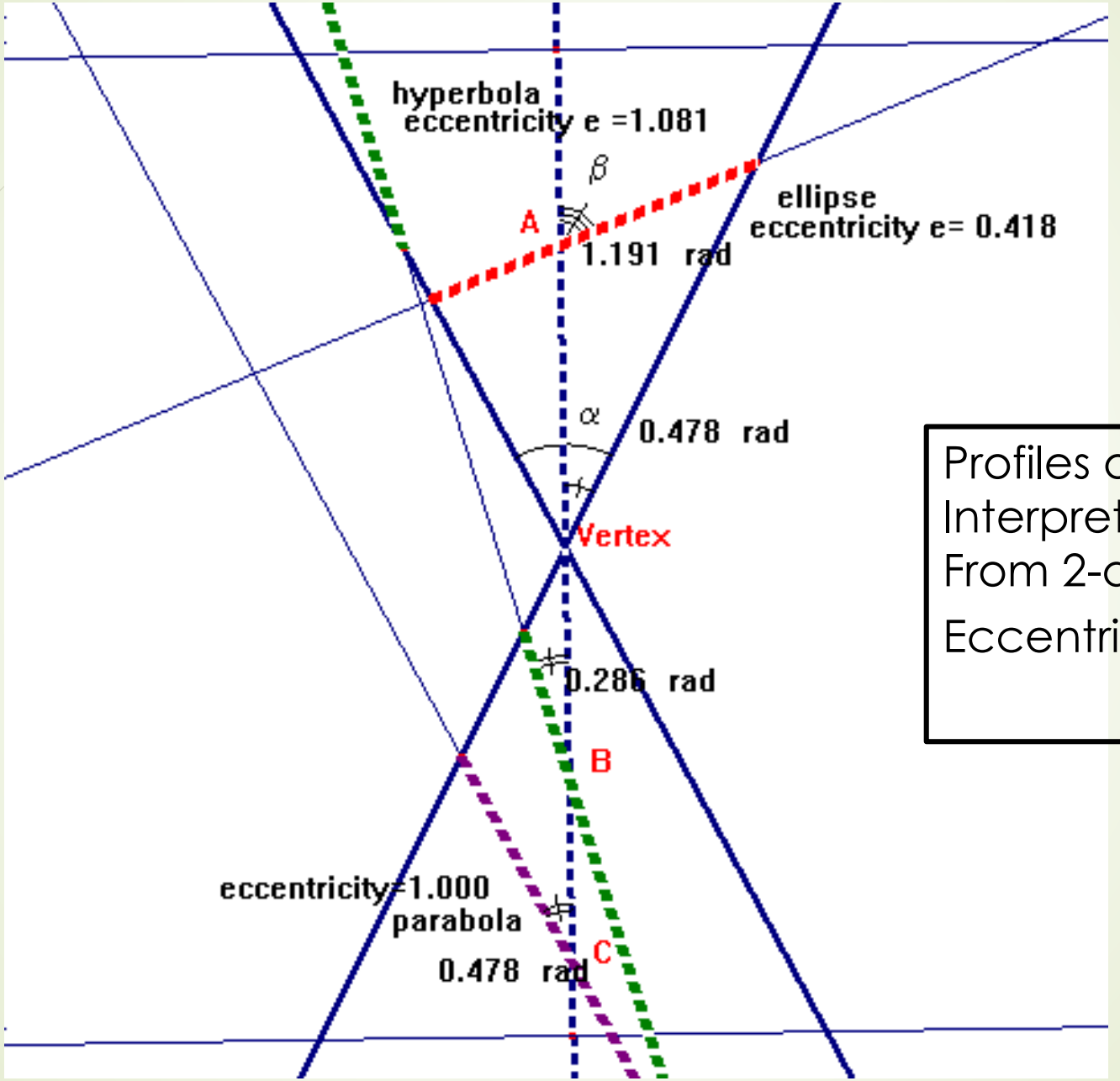
- Technology aided interpretation of the eccentricity of a conic section.

Baldin YY& Furuya, YKS (2001)

CAS graphic representation



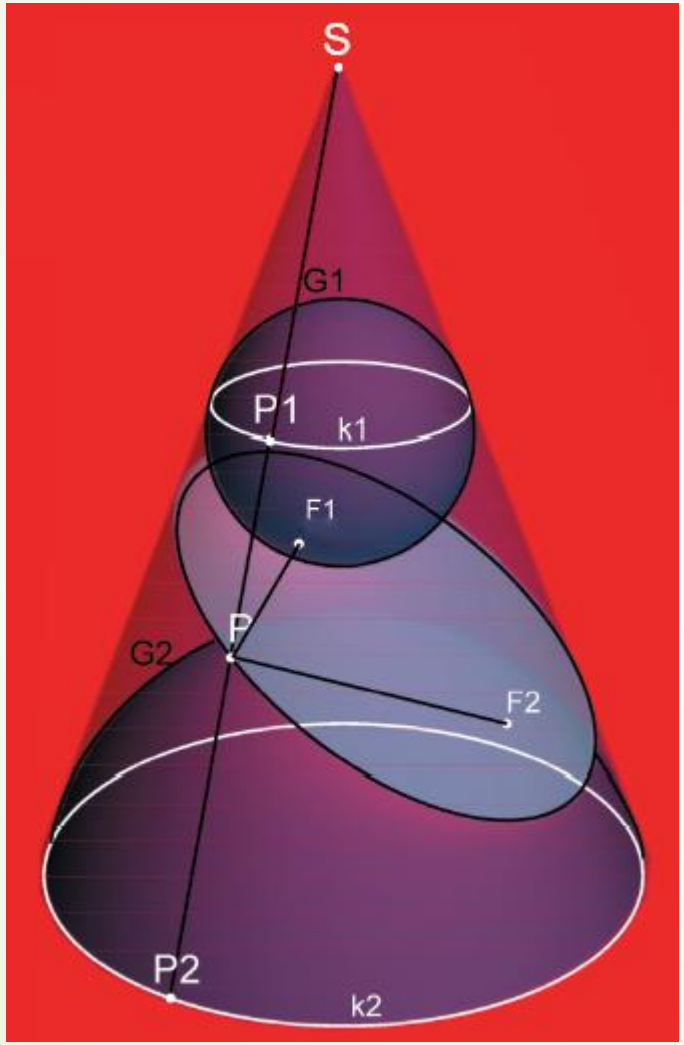




Profiles of conical sections
Interpreting the 3-dim setting
From 2-dim perspective.

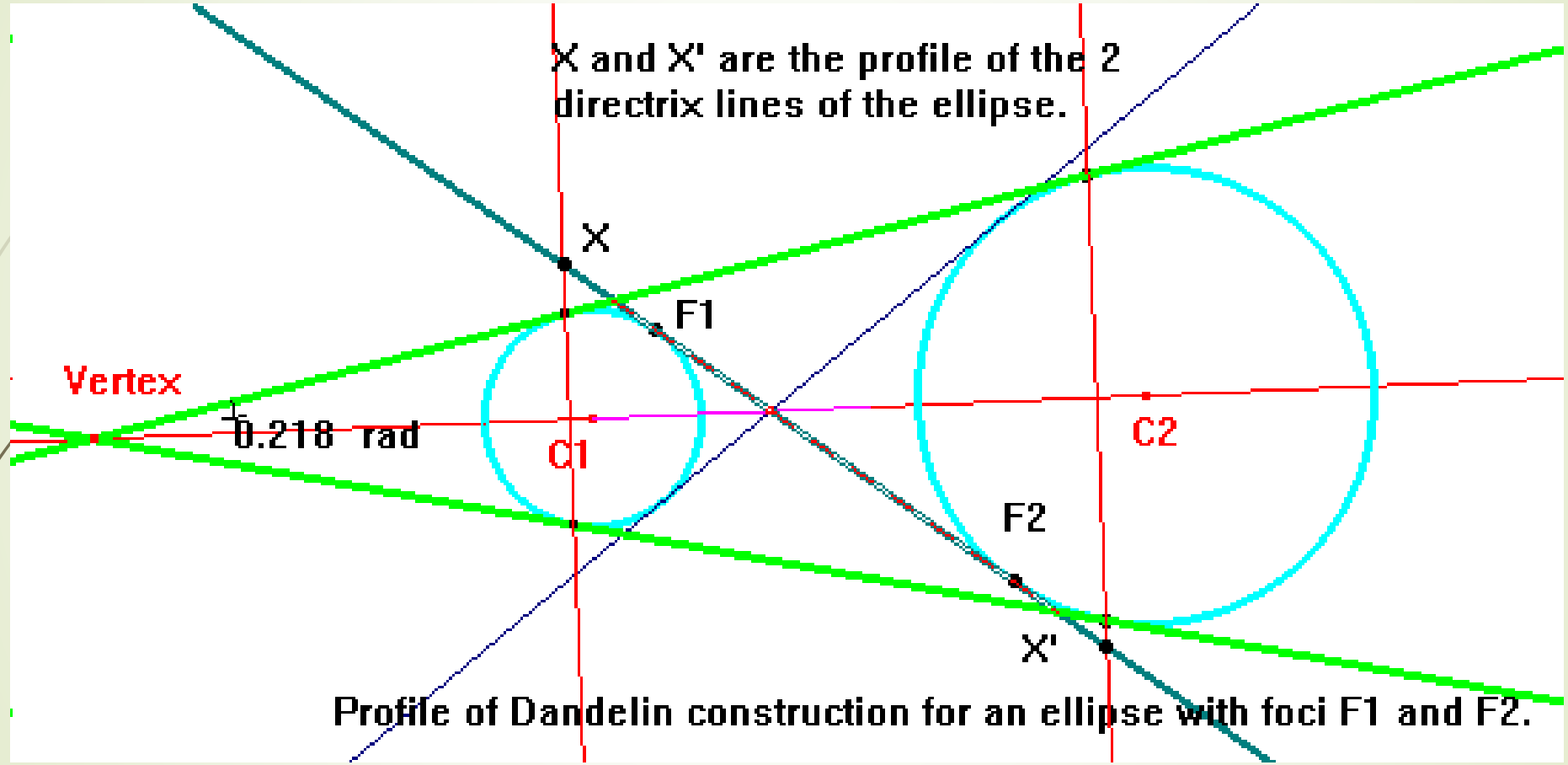
Eccentricity = $\frac{\sin(\propto)}{\sin(\beta)}$

Dandelin Spheres and Conic sections (1822)

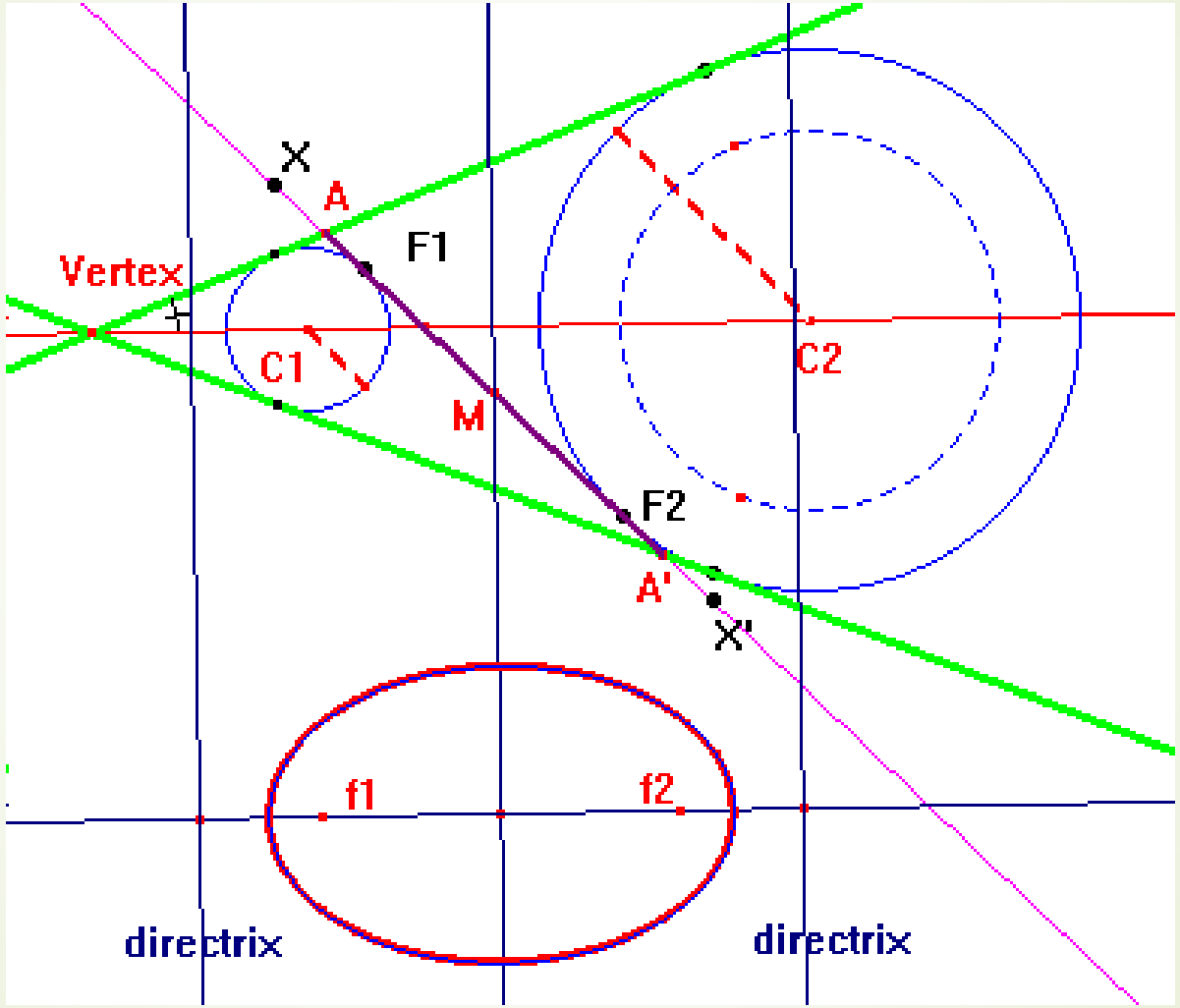


https://en.wikipedia.org/wiki/Dandelin_spheres

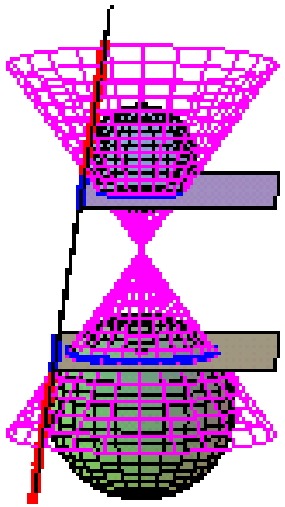
Dandelin Spheres: construction method for the profile of a conical section with DGS

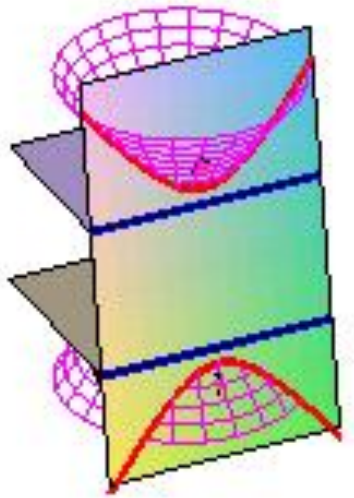


Actual visualization of a conical section on its plane through rotating view in the space.

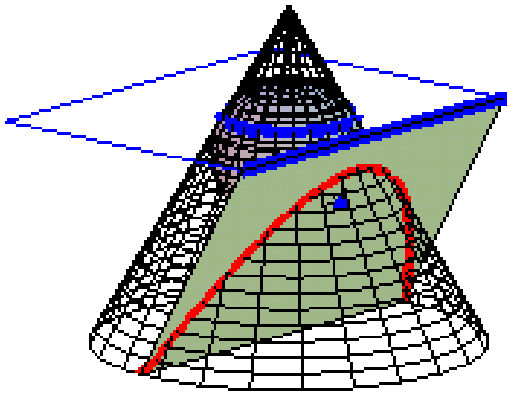


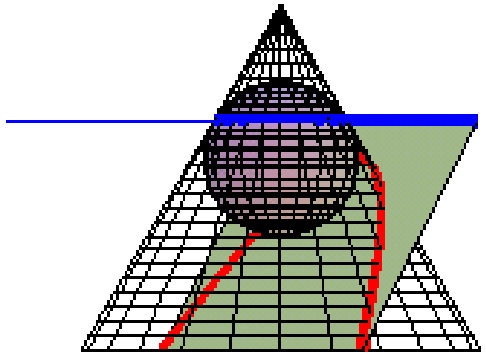
CAS interpretation of Dandelin Spheres in Space
Case of a hyperbola



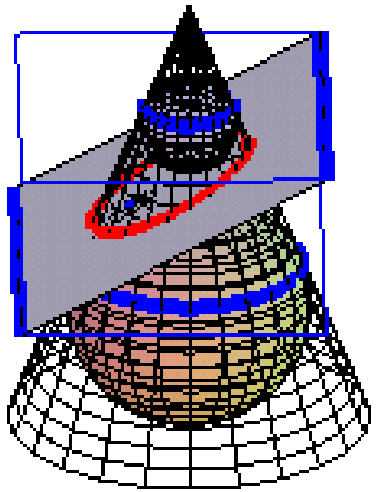


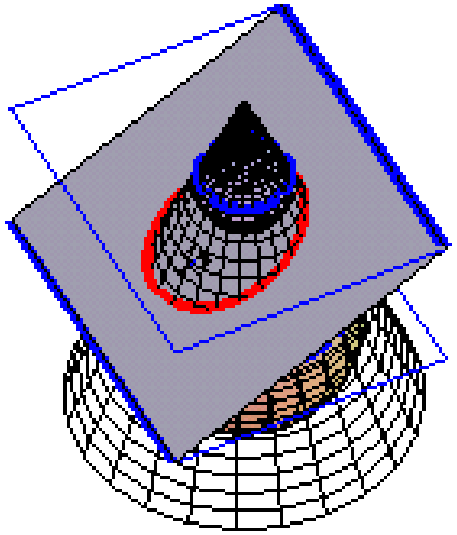
The case of a parabola





The case of a ellipse

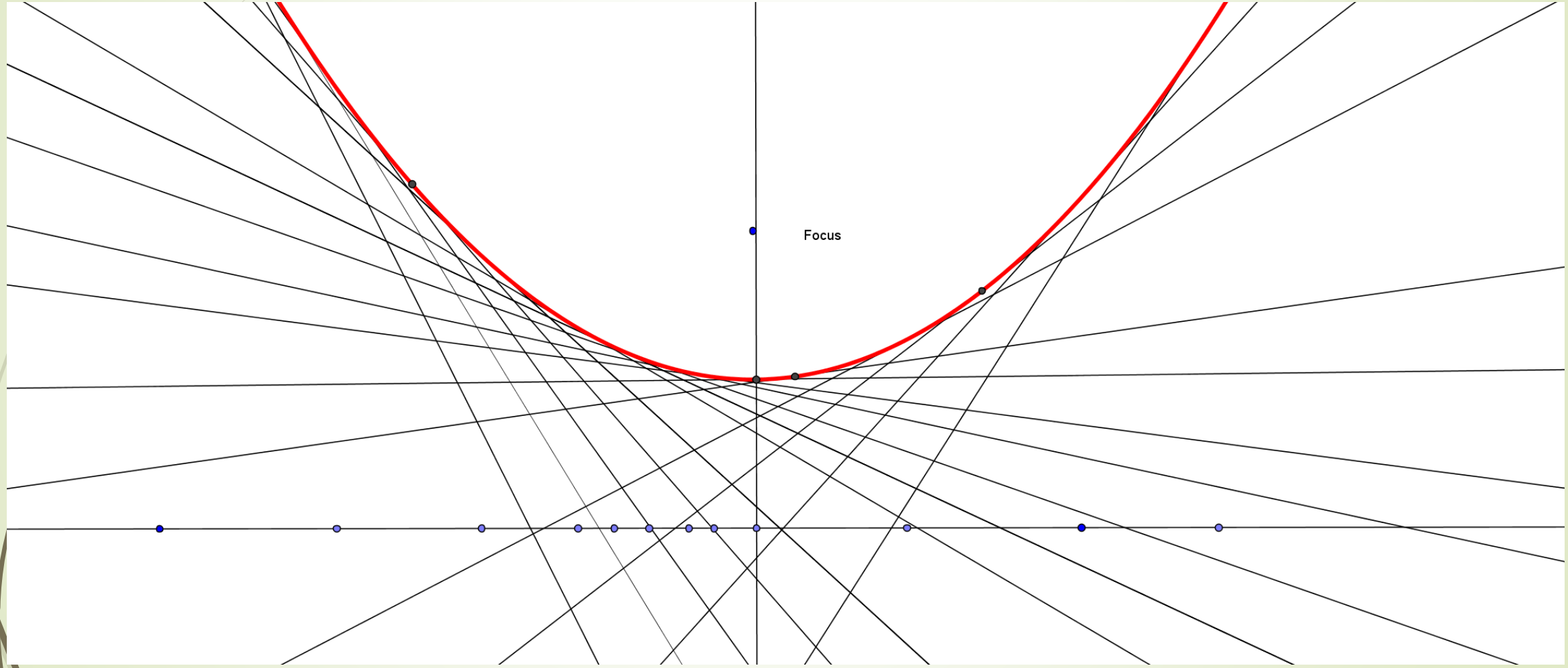




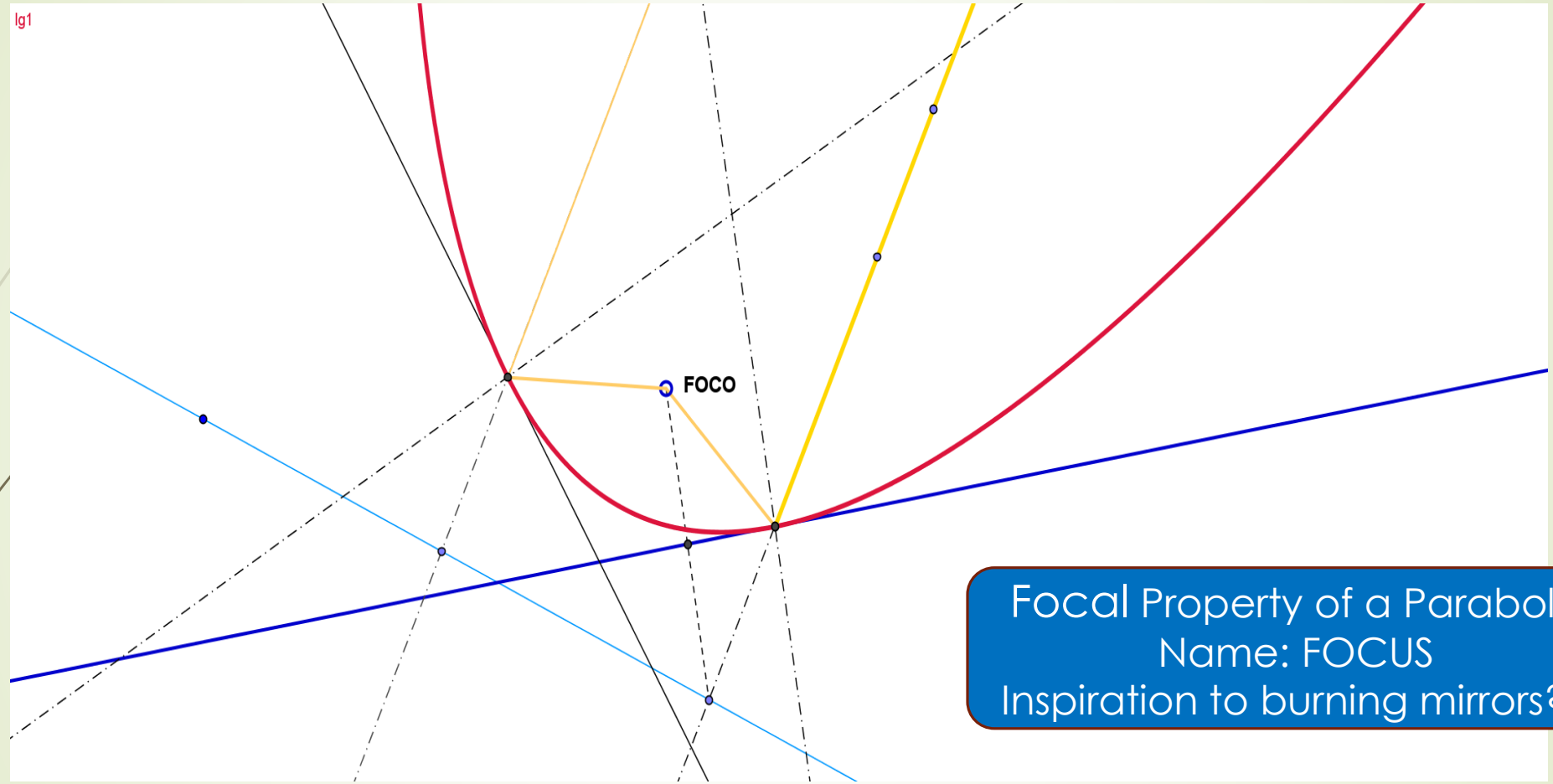
FOR CLASSROOM USE

- Simulating ORIGAMI
- Exploring Focal Properties
- Understanding ageless applications in real world (Historical accomplishments in sciences and technology development for modern facilities and progress for the society)

Paper folding (origami) to understand the geometric property of a parabola as loci. Focus, diretrix, symmetry, concavity, envelope of perpendicular bisectors.



Technology that permits to explore, to conjecture, to discover, to prove (focal property)



Focal Property of a Parabola
Name: FOCUS
Inspiration to burning mirrors??

HISTORICAL IMPORTANCE TO IMPROVE THE QUALITY OF TELESCOPES (FROM EARLY AS 11TH CENTURY) TO GALILEO´S TELESCOPE (16TH CENTURY) TO GREGORY´S PARABOLIC REFLECTOR TELESCOPE AND SUBSEQUENT PROGRESS (NEWTON,HOOKE, CASSEGRAIN). SCIENTIFIC ACCOMPLISHMENTS FROM MATHEMATICS.

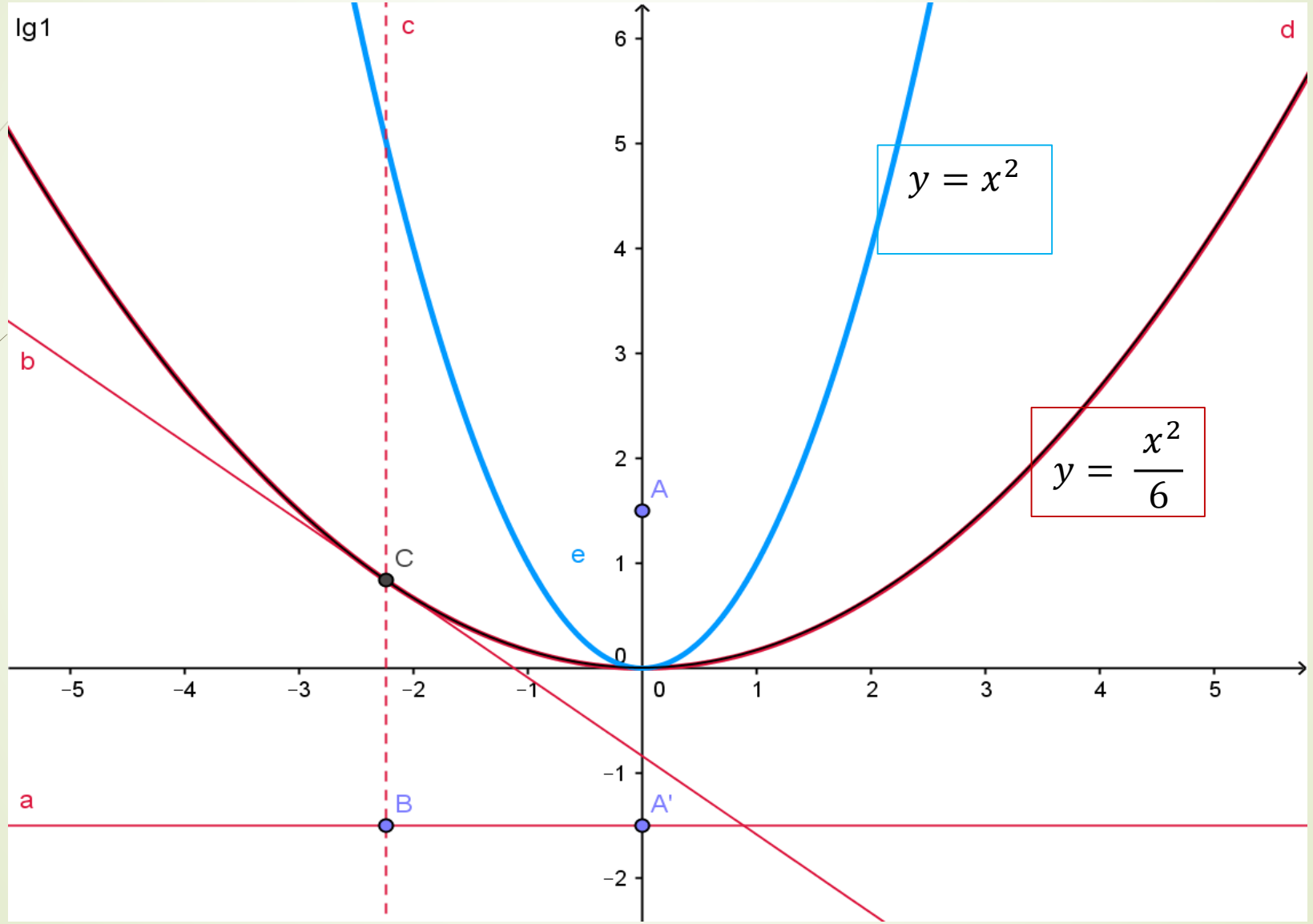
Parabolic Satellite Dish (signal receiver)

Fonte:http://www.floy.com.br/img/class/122011/antenas_parabolicas_22815.gif

More applications for classroom: headlamps of cars
Projects on modern astronomical telescopes.
Trajectory of a thrown object (mechanics principle - gravitation)
Some profiles of suspended bridgeconstruction (and if the extreme points are close?)

CONNECTING THE KNOWLEDGE: GEOMETRY AND ALGEBRA (AFTER DESCARTES)

What has the school content about graphic of a second degree function to do with parabola and FOCUS? Hermeneutics effort to connect the concepts.



Conclusion

- Hermeneutic effort at the service of a better education enables the understanding of mathematics lessons through problem solving activities as a human enterprise and construction of the knowledge as a human heritage through the ages.
- The mission of teacher educators and researchers of teaching mathematics should be focused in providing opportunity to everybody to achieve mathematics literacy through problem solving.
- The effective use of technology is not only a communication device for information, but it should permit to each student trace the paths already walked by others, so he/she can construct own knowledge understanding the perspective of others.
- Lesson Study is grounding methodology to stimulate teachers to become better teachers and researchers of own practice, in a hermeneutic effort for mathematics education.

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Thank you very much for your attention!